Angle Modulated Systems

1. Consider an FM wave
   $f(t) = \cos[2\pi f_c t + \beta_1 \sin 2\pi f_1 t + \beta_2 2\pi f_2 t]$
   The maximum deviation of the instantaneous frequency from the carrier frequency f_c is
   (a) $\beta_1 f_1 + \beta_2 f_2$
   (b) $\beta_1 f_2 + \beta_2 f_1$
   (c) $\beta_1 + \beta_2$
   (d) $f_1 + f_2$

[GATE 2014: 1 Mark]

Soln. The instantaneous value of the angular frequency

$$\boldsymbol{\omega_i = \omega_c + \frac{d}{dt}(\beta_1 \sin 2\pi f_1 t + \beta_2 \sin 2\pi f_2 t)}$$

$$\boldsymbol{\omega_c + 2\pi\beta_1 f_1 \cos 2\pi f_1 t + 2\pi\beta_2 f_2 \cos 2\pi f_2 t}$$

$$\boldsymbol{f_i = f_c + \beta_1 f_1 \cos 2\pi f_1 t + \beta_2 f_2 \cos 2\pi f_2 t}$$

Frequency deviation $(\Delta_f)_{max} = \boldsymbol{\beta_1 f_1 + \beta_2 f_2}$

Option (a)

2. A modulation signal is $y(t) = m(t)\cos(40000\pi t)$, where the baseband signal m(t) has frequency components less than 5 kHz only. The minimum required rate (in kHz) at which y(t) should be sampled to recover m(t) is ___

[GATE 2014: 1 Mark]

Soln. **The minimum sampling rate is twice the maximum frequency called Nyquist rate**

The minimum sampling rate (Nyquist rate) = 10K samples/sec

3. Consider an angle modulation signal $x(t) = 6cos[2\pi \times 10^3 + 2\sin(8000\pi t) + 4\cos(8000\pi t)]V$. The average power of $x(t)$ is
   (a) 10 W
   (b) 18 W
   (c) 20 W
   (d) 28 W

[GATE 2010: 1 Mark]

Soln. The average power of an angle modulated signal is

$$\frac{A_c^2}{2} = \frac{6^2}{2}$$

=18 W

Option (b)

4. A modulation signal is given by $s(t) = e^{-at}\cos[(\omega_c + \Delta_\omega)t]\,u(t)$, where,$\omega_c$ and $\Delta\omega$ are positive constants, and $\omega_c \gg \Delta\omega$. The complex envelope of s(t) is given by
   (a) $\exp(-at)\,exp[j(\omega_c + \Delta\omega)]u(t)$
   (b) $\exp(-at)\,\exp(j\Delta\omega t)\,u(t)$
   (c) $exp(j\Delta\omega t)u(t)$
   (d) $exp[(j\omega_c + \Delta\omega)t]$

[GATE 1999: 1 Mark]

Soln. $s(t) = e^{-at}\cos[(\omega_c + \Delta_\omega)t]u(t)$

Complex envelope $\widetilde{s(t)} = s(t)e^{-j\omega ct}$

$$= \left[e^{-at}e^{j(\omega_c+\Delta_\omega)t}.u(t)\right]e^{-j\omega_c t}$$

$$= e^{-at}e^{j\Delta_\omega t}u(t)$$

Option (b)

5. A 10 MHz carrier is frequency modulated by a sinusoidal signal of 500 Hz, the maximum frequency deviation being 50 KHz. The bandwidth required. as given by the Carson's rule is _______

[GATE 1994: 1 Mark]

Soln. By carson's rule

$$\boldsymbol{BW = 2(\Delta_f + f_m)}$$

$$\boldsymbol{= 2(50 + 0.5)}$$

$$\boldsymbol{= 101\ KHz}$$

6. $v(t) = 5[\cos(10^6\pi t) - \sin(10^3\pi t) \times \sin(10^6\pi t)]$ represents
   (a) DSB suppressed carrier signal
   (b) AM signal
   (c) SSB upper sideband signal
   (d) Narrow band FM signal

[GATE 1994: 1 Mark]

Soln. $\boldsymbol{v(t) = 5\cos(10^6\pi t) - \frac{5}{2}\cos(10^6 - 10^3)\,\pi t + \frac{5}{2}\cos(10^6 + 10^3)\pi t}$

Carrier and upper side band are in phase and lower side band is out of phase with carrier

The given signal is narrow band FM signal

Option (d)

7. The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is
   (a) the in-phase component
   (b) the quadrature-component
   (c) zero
   (d) the envelope

[GATE 2003: 1 Mark]

Soln. **The coherent detector rejects the quadrature component of noise therefore noise at the output has in phase component only.**

Option (a)

8. An AM signal and a narrow-band FM signal with identical carriers, modulating signals and modulation indices of 0.1 are added together. The resultant signal can be closely approximated by

(a) Broadband FM (c) DSB-SC

(b) SSB with carrier (d) SSB without carrier

[GATE 2004: 1 Mark]

Soln. $V_{AM}(t) = A\cos\omega_c t + \frac{0.1A}{2}\cos(\omega_c+\omega_m)t + \frac{0.1A}{2}\cos(\omega_c-\omega_m)t$

$$V_{FM}(t)(narrowband)$$

$$= A\cos\omega_c t + \frac{0.1A}{2}\cos(\omega_c+\omega_m)t - \frac{0.1A}{2}\cos(\omega_c-\omega_m)t$$

$$V_{AM}(t) + V_{FM}(t) = 2A\cos\omega_c t + 0.1A\cos(\omega_c+\omega_m)t$$

The resulting signal is SSB with carrier

Option (b)

9. The List-I (lists the attributes) and the List-II (lists of the modulation systems). Match the attribute to the modulation system that best meets it.

List-I

(A) Power efficient transmission of signals

(B) Most bandwidth efficient transmission of voice signals

(C) Simplest receiver structure

(D) Bandwidth efficient transmission of signals with significant dc component

List-II

(1) Conventional AM
(2) FM
(3) VSB
(4) SSB-SC

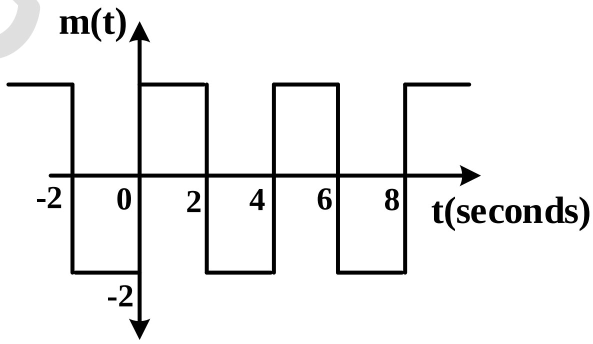
	A	B	C	D
(a)	4	2	1	3
(b)	2	4	1	3
(c)	3	2	1	4
(d)	2	4	3	1

[GATE 2011: 1 Mark]

Soln. FM is the most power efficient transmission of signals AM has the simplest receiver. Vestigial sideband is bandwidth efficient transmission of signals with sufficient dc components. Single sideband, suppressed carrier (SSB-SC) is the most bandwidth efficient transmission of voice signals.

Option (b)

10. The signal m(t) as shown I applied both to a phase modulator (with k_p as the phase constant) and a frequency modulator with (k_f as the frequency constant) having the same carrier frequency
The ratio k_p/k_f (in rad/Hz) for the same maximum phase deviation is



(a) 8π (c) 2π
(b) 4π (d) π

[GATE 2012: 2 Marks]

Soln. For a phase modulator, the instantaneous value of the phase angle ψ_i is equal to phase of an unmodulated carrier $\omega_c(t)$ plus a time varying component proportional to modulation signal m(t)

$$\psi_{PM}(t) = 2\pi f_c t + k_p m(t)$$

Maximum phase deviation $(\psi_{PM})_{max}$

$$= K_p \max m(t) = 2K_p$$

For a frequency modulator, the instantaneous value of the angular frequency

$$\omega_i = \omega_c + 2\pi K_f m(t)$$

The total phase of the FM wave is

$$\psi_{FM} = \int \omega_i dt$$

$$= \omega_c t + 2\pi K_f \int_0^t m(t)dt$$

$$(\psi_{FM})_{max} = 2\pi K_f \int_0^2 2dt$$

$$= 8\pi K_f$$

$$\frac{K_P}{K_f} = \frac{8\pi}{2}$$

$$= \boldsymbol{4\pi}$$

Option (b)

11. Consider the frequency modulated signal

$$10[\cos 2\pi \times 10^5 t + 5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 1000t)]$$

with carrier frequency of 10^5 Hz. The modulation index is

(a) 12.5 (c) 7.5

(b) 10 (d) 5

[GATE 2008: 2 Marks]

Soln. Frequency modulated signal

$$\mathbf{10\cos\left[2\pi \times 10^5 t + 5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 10000t)\right]}$$

The instantaneous value of the angular frequency

$$\boldsymbol{\omega_i = \omega_c + \frac{d}{dt}[5\sin(2\pi \times 1500t) + 7.5\sin(2\pi \times 1000t)]}$$

Frequency deviation

$$\boldsymbol{\Delta_\omega = 5 \times 2\pi \times 1500\cos(2\pi \times 1500) + 7.5 \times 2\pi \times 1000\cos(2\pi \times 1000t)}$$

$$\boldsymbol{(\Delta_\omega)_{max} = 2\pi(7500 + 7500)}$$

Frequency deviation $\boldsymbol{(\delta) = \frac{(\Delta_\omega)_{max}}{2\pi} = 15000Hz}$

Modulation index $\boldsymbol{m_f = \frac{15000}{1500}}$

= 10

Option (b)

12. A message signal with bandwidth 10 KHz is Lower-Side Band SSB modulated with carrier frequency $f_{c_1} = 10^6 Hz$. The resulting signal is then passed through a narrow-band frequency Modulator with carrier frequency $f_{c2} = 10^9 Hz$.
The bandwidth of the output would be

(a) $4 \times 10^4 Hz$
(b) $2 \times 10^6 Hz$
(c) $2 \times 10^9 Hz$
(d) $2 \times 10^{10} Hz$

[GATE 2006: 2 Marks]

Soln. Lower side band frequency $= \mathbf{10^3 - 10}$

$$= \mathbf{990}\ KHz$$

Considering this as the baseband signal, the bandwidth of narrow band FM

$$= \mathbf{2 \times 990}\ KHz$$

$$\approx \mathbf{2} MHz$$

Option (b)

13. A device with input $x(t)$ and output $y(t)$ is characterized by: $y(t) = x^2(t)$. An FM signal with frequency deviation of 90 KHz and modulating signal bandwidth of 5 KHz is applied to this device. The bandwidth of the output signal is

(a) 370 KHz
(b) 190 KHz
(c) 380 KHz
(d) 95 KHz

[GATE 2005: 2 Marks]

Soln. Frequency deviation $\Delta_f = \mathbf{90} KHz$

Modulating signal bandwidth = 5 KMz

When FM signal is applied to doubler frequency deviation doubles.

$$\boldsymbol{B.W = 2(\Delta_f + f_m)}$$

$$= 2(180 + 5)$$

$$= 370\ KHz$$

Option (a)

14. An angle-modulation signal is given by
$s(t) = \cos(2\pi \times 2 \times 10^6 t + 2\pi \times 30 \sin 150t + 2\pi \times 40 \cos 150t)$
The maximum frequency and phase deviations of s(t) are
(a) 10.5KHz, 140π rad
(b) 6 KHz, 80π rad
(c) 10.5 KHz, 100π rad
(d) 7.5 KHz, 100π rad

[GATE 2002: 2 Marks]

Soln. The total phase angle of the carrier

$$\psi = \omega_c t + \theta_0$$

$$\psi = 2\pi \times 2 \times 10^6 + 2\pi \times 30 \sin 150\, t + 2\pi \times 40 \cos 150t$$

Instantaneous value of angular frequency ω_i

$$\omega_i = \omega_c + \frac{d}{dt}(2\pi \times 30 \sin 150t + 2\pi \times 40 \cos 150t)$$

$$= \omega_c + 2\pi \times 30 \times 150 \cos 150t - 2\pi \times 40 \times 150 \sin 150t$$

$$= \omega_c + 2\pi \times 4500 \cos 150t - 2\pi \times 6000 \sin 150\, t$$

Frequency deviation $\Delta_\omega = 2\pi \times 1500[3 \cos 150t - 4 \sin 150t]$

$$= 3000\pi\sqrt{3^2 + 4^2}\ rad/sec$$

$$= 1500\pi\ rad/sec$$

$$= 2\pi \times 7.5K\ rad/sec$$

$$\Delta_f = \frac{\Delta_\omega}{2\pi} = 7.5KHz$$

Phase deviation Δ_ψ is proportional to θ_0

$$\Delta\psi = 2\pi\sqrt{30^2 + 40^2}$$

$$= 2\pi \times 50 = 100\pi\, rad$$

Option (d)

15. In a FM system, a carrier of 100 MHz is modulated by a sinusoidal signal of 5 KHz. The bandwidth by Carson's approximation is 1MHz. If y(t) = (modulated waveform)^3, then by using Carson's approximation, the bandwidth of y(t) around 300 MHz and the spacing of spectral components are, respectively.

(a) 3 MHz, 5 KHz
(b) 1 MHz, 15 KHz
(c) 3 MHz, 15 KHz
(d) 1 MHz, 5 KHz

[GATE 2000: 2 Marks]

Soln. In an FM signal, adjacent spectral components will get separated by modulating frequency $f_m = 5KHz$

$$BW = 2(\Delta_f + f_m) = 1MHz$$

$$\Delta_f + f_m = 500\, KHz$$

$$\Delta_f = 495\, KHz$$

The nth order non-linearity makes the carrier frequency and frequency deviation increased by n-fold, with baseband frequency f_m unchanged.

$$(\Delta_f)new = 3 \times 495$$

$$= 1485\, KHz$$

$$New\ BW = 2(1485 + 5) \times 10^3$$

$$= 2.98\, MHz$$

$$\approx 3\, MHz$$

Option (a)

16. An FM signal with a modulation index 9 is applied to a frequency tripler. The modulation index in the output signal will be

(a) 0
(b) 3
(c) 9
(d) 27

[GATE 1996: 2 Marks]

Soln. The frequency modulation index β is multiplied by n in ntimes frequency multiplier.

$$\boldsymbol{So,\ \ \beta' = 3 \times 9}$$

$$= \mathbf{27}$$

Option (d)

17. A signal $x(t) = 2\cos(\pi.10^4 t)$ volts is applied to an FM modulator with the sensitivity constant of 10 KHz/volt. Then the modulation index of the FM wave is

(a) 4
(b) 2
(c) 4/π
(d) 2/π

[GATE 1989: 2 Marks]

Soln. Modulation index

$$\boldsymbol{\beta = \frac{K_f A_m}{f_m}}$$

$$\boldsymbol{K_f = 10KHz/volt}$$

A_m is the amplitude of modulating signal

f_m is the modulating frequency

$$\boldsymbol{\beta = \frac{10 \times 10^3 \times 2}{\frac{\pi \times 10^4}{2\pi}} = 4}$$

Option (a)

18. A carrier $A_C \cos \omega_c\, t$ is frequency modulated by a signal $E_m \cos \omega_m\, t$.The modulation index is m_f. The expression for the resulting FM signal is

(a) $A_c \cos[\omega_c t + m_f \sin \omega_m t]$

(b) $A_c \cos[\omega_c t + m_f \cos \omega_m t]$

(c) $A_c \cos[\omega_c t + 2\pi\, m_f \sin \omega_m t]$

(d) $A_C \cos\left[\omega_c t + \frac{2\pi m_f E_m}{\omega_m} \cos \omega_m t\right]$

[GATE 1989: 2 Marks]

Soln. The frequency modulated signal

$$\boldsymbol{V_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int m(t)dt\right]}$$

K_f is the frequency sensitivity of the modulator

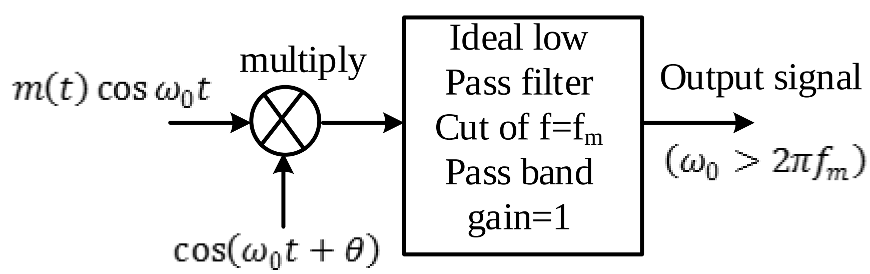
$$\boldsymbol{\int m(t)dt = \int E_m \cos \omega_m t\; dt = \frac{E_m \sin \omega_m t}{\omega_m}}$$

$$\boldsymbol{V_{FM}(t) = A_c \cos\left[\omega_c t + \frac{K_f E_m}{\omega_m} \sin \omega_m t\right]}$$

$\boldsymbol{= A_c \cos[\omega_c t + m_f \sin \omega_m t]}$**where m_f is the modulation index**

Option (a)

19. A message m(t) bandlimited to the frequency f_m has a power of P_m. The power of the output signal in the figure is



(a) $\frac{P_m \cos\theta}{2}$

(b) $\frac{P_m}{4}$

(c) $\frac{P_m sin^2\theta}{4}$

(d) $\frac{P_m cos^2\theta}{4}$

[GATE 2000: 2 Marks]

Soln. Output of the multiplier $= \boldsymbol{m(t)\cos\omega_0 t\, cos(\omega_0 t + \theta)}$

$$= \frac{m(t)}{2}[\cos(2\omega_0 t + \theta) + \cos\theta]$$

Output of LPF $V_0(t) = \frac{m(t)}{2}\cos\theta$

$$= \frac{1}{2}\cos\theta\, m(t)$$

Power of output signal

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T V_0^2(t)dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int \frac{Cos^2\theta}{4} m^2(t)dt$$

$$= \frac{cos^2\theta}{4}\lim_{T\to\infty}\frac{1}{T}\int_0^T m^2(t)dt$$

$$= \frac{cos^2\theta}{4}P_m$$

Option (d)

20. c(t) and m(t) are used to generate an FM signal. If the peak frequency deviation of the generated FM signal is three times the transmission bandwidth of the AM signal , then the coefficient of the term

$5\cos[2\pi(1008 \times 10^3 t)]$ in the FM signal (in terms of the Bessel coefficients) is

(a) $5J_4(3)$

(b) $\frac{5}{2}J_8(3)$

(c) $\frac{5}{2}J_8(4)$

(d) $5J_4(6)$

[GATE 2003: 2 Marks]

Soln.

$$V_{FM}(t) = \sum_{n=-\infty}^{\infty} AJ_n(m_f)\cos(\omega_c + n\omega_m)t$$

Peak frequency deviation of FM signal is three times the bandwidth of AM signal

$$\delta_f = 3 \times 2f_m = 6f_m$$

Modulation index

$$m_f = \frac{\delta_f}{f_m} = \frac{6f_m}{f_m} = 6$$

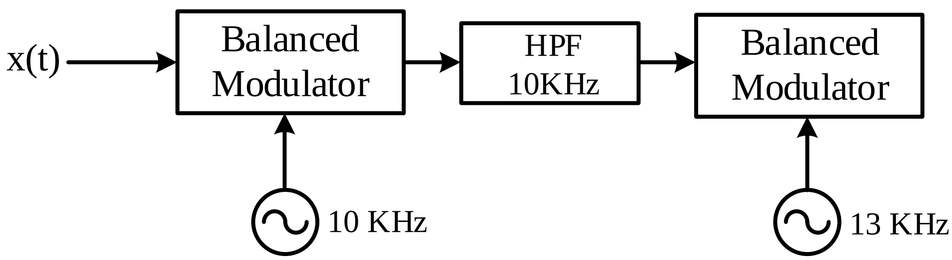
$$5\cos[2\pi(1008 \times 10^3 t)] = 5\cos[2\pi(1000 + 4 \times 2) \times 10^3]$$

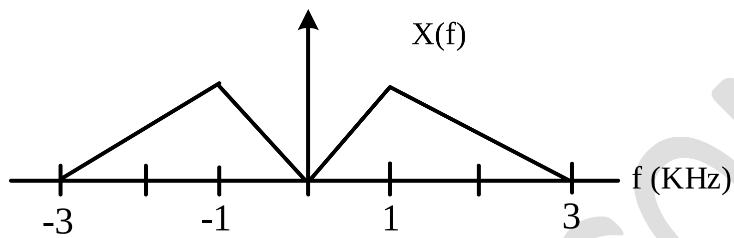
$$n = 4$$

The required coefficient is $5J_4(6)$

Option (d)

21. Consider a system shown in the figure. Let $X(f)$ *and* $Y(f)$ denote the Fourier transforms of $x(t)$ *and* $y(t)$ respectively. The ideal HPF has the cutoff frequency 10 KHz.





The positive frequencies where Y(f) has spectral peaks are

(a) 1 KHz and 24 KHz
(b) 2 KHz and 24 KHz
(c) 1 KHz and 14 KHz
(d) 2 KHz and 14 KHz

[GATE 2004: 2 Marks]

Soln. Input signal x(f) has the peaks at 1KHz and -1Mhz.

The output of balanced modulator will have peaks at

$$\boldsymbol{f_c \pm 1, f_c \pm (-1) f_c \pm 1 = 10 \pm 1}$$

$$\boldsymbol{= 11 \; and \; 9 \; KHz}$$

$$\boldsymbol{f_c \pm (-1) = 10 \pm (-1) = 9 \; KHz \; and \; 11 \; KHz}$$

9 MHz will be filtered out by the HPF

After passing through 13 KHz balanced modulator, the signal will have $\boldsymbol{13 \pm 11}$ frequencies.

$$\boldsymbol{y(f) = 24 \; K \; and \; 2 \; K}$$

Option (b)

Common Data for Questions 22 & 23

Consider the following Amplitude Modulated (AM) signal,

Where $f_m < B \quad X_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t$

22. The average side-band power for the AM signal given above is

(a) 25 (c) 6.25

(b) 12.5 (d) 3.125

[GATE 2006: 2 Marks]

Soln. The average sideband power for the AM signal is

$$P_{SB} = P_c \frac{ma^2}{2}$$

$$P_c \rightarrow carrier\ power$$

$$m_a \rightarrow modulation\ index$$

$$P_c = \frac{Ac^2}{2} = \frac{10^2}{2}$$

$$= 50\omega$$

$$m_a = 0.5$$

So,

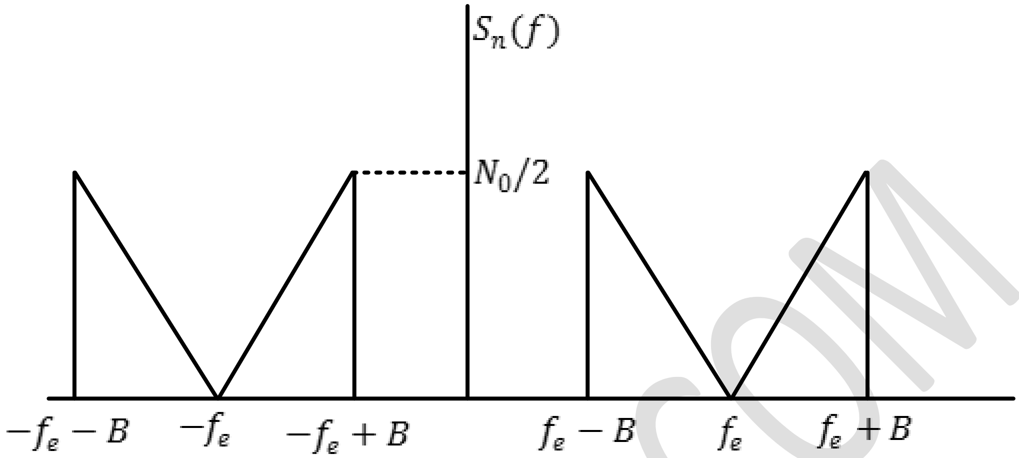
$$P_{SB} = 50 \frac{(0.5)^2}{2}$$

$$= \frac{50 \times 0.25}{2}$$

$$= 6.25\ watts$$

Option (c)

23. The AM signal gets added to a noise with Power Spectral Density $S_n(f)$ given in the figure below. The ratio of average sideband power to mean noise power would be:



(a) $\frac{25}{8N_0B}$

(b) $\frac{25}{4N_0B}$

(c) $\frac{25}{2N_0B}$

(d) $\frac{25}{N_{0B}}$

[GATE 2006: Marks]

Soln. The AM signal gets added to a noise with spectral density $s_n(f)$

The noise power

$$P_T = \int_{-\infty}^{\infty} s_n(f)df$$

$$= 2\int_{0}^{\infty} s_n(f)df$$

$$\textbf{Noise power} = 4\left[\frac{1}{2} \times \frac{B}{1} \times \frac{N_0}{2}\right]$$

$$= N_0B$$

$$\textbf{Power in sidebands } P_{SB} = \frac{25}{4}\ watts$$

$$\frac{P_{SB}}{noise\ power} = \frac{25}{4N_0B} \qquad option\ (b)$$