Pulse Modulation Systems

(a) Sampling

1. A bandlimited signal is sampled at the Nyquist rate. The signal can be recovered by passing the samples through
   (a) an RC filter
   (b) an envelope detector
   (c) a PLL
   (d) an ideal low-pass filter with the appropriate bandwidth

[GATE 2001: 1 Mark]

Soln. A continuous time signal is sampled at Nyquist rate

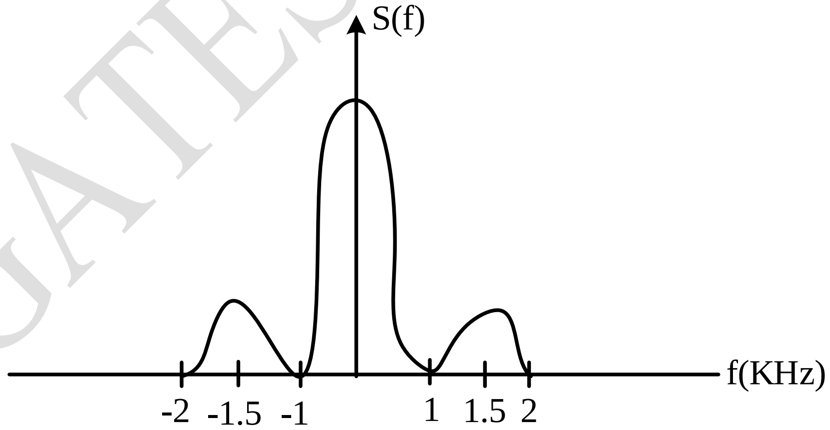
i.e. $f_s = 2f_m$ Samples/sec

It can be fully recovered.

The spectrum F(ω) repeats periodically without overlap. The signal can be recovered by passing sampled signal through a low pass filter with sharp cut- off at frequency f_m .

Option (d)

2. A deterministic signal has the power spectrum given in figure. The minimum sampling rate needed to completely represent signal is



(a) 1 KHz
(b) 2 KHz
(c) 3 KHz
(d) none of these

[GATE 1997: 1 Mark]

Soln. The given power spectrum has main lobe and other side lobes. We observe that the maximum frequency of main lobe is 1KHz.

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practically about 90% of the total signal lies in the major lobe. So maximum frequency of the signal can be taken as 1 KHz.

Minimum sampling rate (Nyquist rate) = 2f_m

Nyquist rate = $2 \times 1KHz = 2KHz$

Option (b)

3. A signal has frequency components from 300 Hz to 1.8 KHz. The minimum possible rate at which the signal has to be sampled is ---

[GATE 1991: 2 Marks]

Soln. This is the case of band pass sampling

$$f_H = 1800\ Hz$$

$$f_L = 300\ Hz$$

Bandwidth $= f_H - f_L = 1800 - 300 = 1500\ Hz$

Now, $m = \frac{f_H}{BW} = \frac{1800}{1500} = 1.2 \quad , \ Integer = 1.0$

$$(f_s)_{min} = \frac{2f_H}{m} = \frac{2\times1800}{1} = 3600\ samples/sec$$

Ans. 3600 samples/sec

4. Flat top sampling of low pass signals
   (a) gives rise to aperture effect
   (b) implies oversampling
   (c) leads to aliasing
   (d) introducing delay distortion

[GATE 1998: 1 Mark]

Soln. Flat top sampling of low pass signal has the spectrum of sinc function where amplitude of high frequency components is reduced. This affect is called Aperture effect. An aperture refers to the sampling process as window (aperture) of finite time width through which signal voltage is observed.

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This can be reduced by either increasing the sampling frequency or reducing sampling aperture width. As width increases loss increases

Option (a)

5. Increased pulse width in the flat top sampling leads to
   (a) attenuation of high frequencies in reproduction
   (b) attenuation of low frequencies in reproduction
   (c) greater aliasing errors in reproduction
   (d) no harmful effects in reproduction

[GATE 1994: 1 Mark]

Soln. For flat top sampling the spectrum is of sinc function, where the amplitude of high frequency component is reduced. This effect is called aperture effect.

As the sampling aperture increases i.e. increase of pulse width ------ there will be more loss in high frequency components.

Option (a)

6. A 1.0 KHz signal is flat top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100 Hz, then the output of the filter contains
   (a) only 800 Hz component
   (b) 800 Hz and 900 Hz components
   (c) 800 Hz and 1000 Hz components
   (d) 800 Hz, 900 Hz and 100 Hz components

[GATE 1995: 1 Mark]

Soln. Given

$$f_m = 1KHz$$

$$f_s = 1.8\ k\ samples/sec$$

The frequency components in the sampled signal are

$$nf_s \pm f_m$$

For $n = 0, \quad f_m = 1KHz = 1000Hz$

$n = 1, \quad frequencies\ \ are\ \ 1.8 \pm 1\ i.e. 800Hz\ \&\ 2800\ Hz$

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$\boldsymbol{n = 2, \quad frequencies \;\; are \; 3.6 \; \pm 1 \; i.e. 2600Hz \; \& \; 4600 \; Hz}$

Cutoff frequency of LPF 1100 Hz

So, 800 Hz & 1000 Hz components will appear at the output

Option (c)

7. The Nyquist sampling interval, for the signal $\sin c(700\,t) + \sin c(500\,t)$ is

(a) $\frac{1}{350}\,sec$
(b) $\frac{\pi}{350}\,sec$
(c) $\frac{1}{700}\,sec$
(d) $\frac{\pi}{175}\,sec$

[GATE 2001: 2 Marks]

Soln. Signal is given as

$$\boldsymbol{\sin c\,(700t) + \sin c\,(500t)}$$

$$\boldsymbol{= \frac{\sin(700\pi t)}{700\pi t} + \frac{\sin(500\pi t)}{500\pi t}}$$

The maximum frequency component is

$$\boldsymbol{2\pi f_m = 700\pi}$$

Or $\boldsymbol{f_m = 350Hz}$

The Nyquist rate is $\boldsymbol{f_s = 2f_m}$

$$\boldsymbol{= 2 \times 350\;Hz}$$

$$\boldsymbol{= 700\;Hz}$$

Sampling interval $\boldsymbol{\left(\frac{1}{f_s}\right) = \frac{1}{700}\,sec}$

Option (c)

8. The Nyquist sampling frequency (in Hz) of a signal given by $6 \times 10^4 \sin c^2(400t) * 10^6 \sin c^3(100t)$ is

(a) 200
(b) 300
(c) 1500
(d) 1000

[GATE 1999: 2 Marks]

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Soln. **The signal given is**

$$6 \times 10^4 \sin c^2 (400t) * 10^6 \sin c^3 (100t)$$

Let the given function is f(t) and

$$f_1(t) = 6 \times 10^4 \sin c^2 (400t)$$

$$f_2(t) = 10^6 \sin c^3 (100t)$$

We know that

$$f_1(t) * f_2(t) \rightleftharpoons F_1(\omega).F_2(\omega)$$

Convolution in time domains is product is frequency domain

Bandwidth of

$$F_1(\omega) = 2 \times 400\, rad/sec = 800\, rad/\sec\, or\, 400\, Hz$$

Due to sin c^2 term

Bandwidth of

$$F_2(\omega) = 3 \times 100 = 300\, rad/\sec\, or\, 150Hz$$

Due to sin c^2 term

Then $F_1(\omega).F_2(\omega)$ will have bandwidth of 150Hz only (since product of two spectrums)

Sampling frequency is $2 \times 150Hz = 300Hz$

Option (b)

9. Four independent massages have bandwidths of 100 Hz. 100 Hz, 200 Hz and 400 Hz, respectively. Each is sampled at the Nyquist rate, and the samples are Time Division Multiplexed (TDM) and transmitted. The transmitted rate (in Hz) is

(a) 1600
(b) 800
(c) 400
(d) 200

[GATE 1999: 2 Marks]

Soln. **Four messages are sampled at the Nyquist rate**

I – message $f_{s1} = 200Hz$

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II – message $f_{s2} = 200Hz$

III – message $f_{s3} = 400Hz$

IV – message $f_{s4} = 800Hz$

So there are 1600 samples in 1 sec

So, the speed of commutator is 1600 samples per sec

Option (a)

10. A signal $x(t) = 100\cos(24\pi \times 10^3)\, t$ is ideally sampled with a sampling period of 50 μsec and then passed through an ideal low pass filter with cutoff frequency of 15 KHz. Which of the following frequency is/are per present at the filter output?
(a) 12 KHz only
(b) 8 KHz only
(c) 12 KHz and 9 KHz
(d) 12 KHz and 8 KHz

[GATE 2002: 2 Marks]

Soln. Signal frequency $\frac{24\pi\times10^3}{2\pi} = 12\ KHz$

Sampling period $T_S = 50\ \mu sec$

So $f_s = \frac{1}{T_S} = \frac{1}{50\times10^{-6}} = 20\ KHz$

After sampling, signal will have frequency f_m and $f_s \pm f_m$

i.e. 12 KHz and 20 KHz – 12 KHz = 8 KHz

so, after filter the frequency present will be 12 KHz and 8 KHz

Option (d)

Note that sampling frequency (20 KHz) is less than 2 f_m (24 KHz) Nyquist rate, so there will be aliasing error.

11. Let $x(t) = 2\cos(800\pi t) + \cos(1400\pi t)$. $x(t)$ is sampled with the rectangular pulse train shown in the figure. The only spectral components

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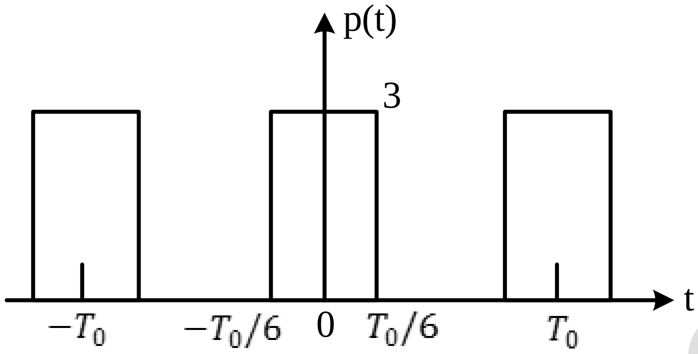
(in KHz) present in the sampled signal in the frequency range 2.5 KHz to 3.5 KHz are



$T = 10^{-3} sec$

(a) 2.7, 3.4

(b) 3.3, 3.6

(c) 2.6, 2.7, 3.3, 3.4, 3.6

(d) 2.7, 3.3

[GATE 2003: 2 Marks]

Soln. Given $T = 10^{-3}$ so frequency $= \frac{1}{10^{-3}} = 1\ KHz$ of pulse train.

To determine frequency components in pulse train we find Fourier series Coefficient C_n

$$C_n = \frac{1}{T_0}\int_{-T_0/6}^{T_0/6} Ae^{-jn\omega_0 t}dt = \frac{1}{T_0}\cdot\frac{A}{(-jn\omega_0)}\left[e^{-jn\omega_0 t}\right]_{-T_0/6}^{T_0/6}$$

$$= \frac{A}{T_0(-jn\omega_0)}\left[e^{-jn\omega_0 T_0/6} - e^{jn\omega_0 T_0/6}\right]$$

$$C_n = \frac{A.2.j}{T_0(jn\omega_0)}\sin\left(\frac{n\pi}{3}\right) = \frac{A}{n\pi}\sin\left(\frac{n\pi}{3}\right)$$

From C_n we see that harmonics present are 1,2,3,4,5,7--------

So p(t) has 1 KHz, 2 KHz, 4 KHz-------

The signal has frequency components 0.4 KHz and 0.7 KHz

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The sampled signal will be $x(t) \times p(t)$ will have

1 ± 0.4 and 1 ± 0.7 KHz

2 ± 0.4 and 2 ± 0.7 KHz

3 ± 0.4 and 3 ± 0.7 KHz etc

In the frequency range of 2.5 KHz to 3.5 KHz

The frequency present are

2 + 0.7 = 2.7 KHz

4 – 0.7 = 3.3 KHz

Option (d)

12. The Nyquist sampling rate for the signal $s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$ is given by

(a) 400 Hz
(b) 600 Hz
(c) 1200 Hz
(d) 1400 Hz

[GATE 2010: 2 Marks]

Soln. Given

$$s(t) = \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t}$$

$$= \frac{1}{2}\left[\frac{2\sin(500\pi t).\sin(700\pi t)}{\pi^2 t^2}\right]$$

$$= \frac{1}{2}[\cos(700\pi t - 500\pi t) - \cos(700\pi t + 500\pi t)]$$

$$= \frac{1}{2\pi^2 t^2}[\cos(200\pi t) - \cos(1200\pi t)]$$

Maximum frequency component

$$f_m = \frac{1200\pi}{2\pi} = 600Hz$$

Nyquist sampling rate

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$$f_{s_{min}} = 2f_m = 1200Hz$$

Option (c)

13. Consider a sample signal

$$y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)$$

Where

$x(t) = 10\cos(8\pi \times 10^3)t \; and \; T_S = 100\,\mu\,sec$
When y(t) is passed through an ideal low-pass filter with a cutoff frequency of 5 kHz, the output of the filter is
(a) $5 \times 10^{-6}\cos(8\pi \times 10^3)t$
(b) $5 \times 10^{-5}\cos(8\pi \times 10^3)t$
(c) $5 \times 10^{-1}\cos(8\pi \times 10^3)t$
(d) $100\cos(8\pi \times 10^3)t$

[GATE 2002: 1 Mark]

Soln. Given, the sample signal

$$\boldsymbol{y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S)}$$

$$\boldsymbol{x(t) = 10\cos(8\pi \times 10^3)\, t}$$

$$\boldsymbol{T_S = 100\mu\text{sec}}$$

Note, Fourier series expansion of impulse train is

$$\boldsymbol{\delta_{TS}(t) = \frac{1}{T_S}[1 + 2\cos\omega_s t + 2\cos 2\omega_s t \; + - - -]}$$

Sampled signal can be written as

$$\boldsymbol{5 \times 10^{-6}.\, x(t).\frac{1}{T_S}[1 + 2\cos\omega_s t + - - -]}$$

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$$= \frac{5 \times 10^{-6}}{100 \times 10^{-6}} . 10 \cos(8\pi \times 10^3) t \, [1 + 2 \cos \omega_s t + \,--]$$

$$= 5 \times 10^{-1} \cos(8\pi \times 10^3) \, t[1 + 2 \cos \omega_s t + - \,--]$$

It is passed through 5 KHz filter then the output will be

$$5 \times 10^3 \cos(8\pi \times 10^3) \, t$$

Option (c)

14. If E_b, the energy per bit of a binary digital signal, is 10^{-5} watt-sec and the one-sided power spectral density of the white noise, $N_0 = 10^{-6}$ W/Hz, then the output SNR of the matched filter is

(a) 26 dB
(b) 10 dB
(c) 20 dB
(d) 13 dB

[GATE 2003: 2 Marks]

Soln. Given

$$E_b = 10^{-5} \, watt \; sec$$

$$N_0 = 10^{-6} \, W/Hz$$

SNR of matched filter

$$= \frac{E_b}{N_0/2} = \frac{10^{-5}}{10^{-6}/2} = \frac{2 \times 10^{-5}}{10^{-6}} = 20$$

$$(SNR)_{dB} = 10 \log 20 = 13dB$$

Option (d)

15. The transfer function of a zero-order hold is

(a) $\frac{1 - exp(-T_S)}{S}$

(b) $\frac{1}{S}$

(c) 1

(d) $\frac{1 - exp(-T_S)}{S}$

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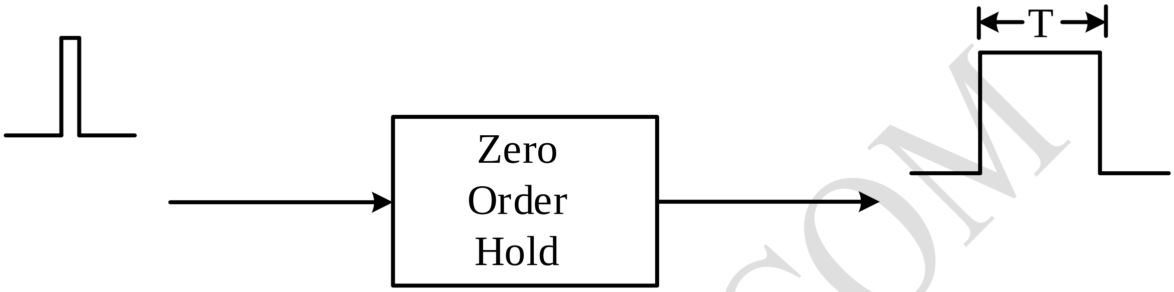
[GATE 1988: 2 Marks]

Soln. **Zero order holding circuit holds the input signal value for a period of T, for an input of short duration $\delta(t)$ it produces an output pulse duration T.**



Input $x(t) = \delta(t) \quad so \quad X(s) = 1$

Output $y(t) = u(t) - u(t - T)$

$$So, \; y(s) = \frac{1}{s} - \frac{e^{-Ts}}{s} = \frac{1 - e^{-Ts}}{s}$$

$$H(s) = \frac{y(s)}{X(s)} = \frac{1 - e^{-Ts}}{s}$$

Option (a)

(b) Pulse Code Modulation (PCM)

16. An analog voltage in the range 0 to 8 V is divided in 16 equal intervals for conversion to 4-bit digital output. The maximum quantization error (in V) is _____________

[GATE 2014: 1 Mark]

Soln. Given,

Dynamic range or voltage range = 0 to 8 V

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Number of levels = 16

Maximum quantization error

$$Q_e = \frac{step\ size\ (\Delta)}{2}$$

Where

$$\Delta = \frac{Dynamic\ range}{L}$$

$$= \frac{8}{2^4} = \frac{8}{16} = 0.5$$

$$Q_e = \frac{0.5}{2} = 0.25\ V$$

Quantization error is 0.25 V

17. Compression in PCM refers to relative compression of
    (a) higher signal amplitudes
    (b) lower signal amplitudes
    (c) lower signal frequencies
    (d) higher signal frequencies

[GATE 1999: 1 Mark]

Soln. In PCM, Companding results in making SNR uniform irrespective of signal amplitude level.

In formed from two words 'compressing' and 'expanding'

In PCM, analog signal values are rounded on a non-linear scale. The data is compressed before it is sent and then expanded at the receiving end using same non-linear scale.

So right option is compression of higher signal amplitudes

Option (a)

18. The line code that has zero dc component for pulse transmission of random binary data is

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(a) non-return to zero (NRZ)
(b) return to zero (RN)
(c) alternate mark inversion (AM)
(d) none of the above

[GATE 1997: 1 Mark]

Soln. There are two types coding

- **Source coding techniques are used in PCM and DM. In this analog signal is converted to digital i.e. train of binary digits**
- **Line coding converts stream of binary digits into a formal or code which is more suitable for transmission over a cable or any other medium**

Alternate mark inversion (AMI) code has zero dc component for pulse transmission of random binary data

Option (c)

19. The signal to quantization noise ratio in an n-bit PCM system
(a) depends upon the sampling frequency employed
(b) is independent of the value of 'n'
(c) increasing with increasing value of 'n'
(d) decreases with the increasing value of 'n'

[GATE 1995: 1 Mark]

Soln. Signal to quantization noise in an n bit PCM is

$$\left(\frac{S}{N_q}\right) = (SQNR) = \frac{3}{2}.2^{2n} ------(1A)$$

Where n is number of bits in the word of binary PCM.

It can be written in dBs

$$(SQNR)_{dB} = \left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6n ----(1B)$$

From above equations (S/N) increases with n

Option (c)

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20. If the number bits per sample in a PCM system is increased from and n to (n+1), the improvement is signal to quantization nose ratio will be

(a) 3 dB
(b) 6 dB
(c) 2 n dB
(d) n dB

[GATE 1995: 1 Mark]

Soln. Note $\left(\frac{S}{N_q}\right)_{dB} = (1.76 + 6n)_{dB}$

$$(SQNR)_1 = 1.76 + 6n$$

$$(SQNR)_2 = 1.76 + 6n(n + 1) = 1.76 + 6n + 6$$

$$(SQNR)_2 - (SQNR)_1 = 1.76 + 6n + 6 - 1.76 - 6n = 6dB$$

So for every one bit increase in bits per sample will result is 6 dB improvement in signal to quantization ratio

Option (b)

21. The bandwidth required for the transmission of a PCM signal increases by a factor of _____________ when the number of quantization levels is increased from 4 to 64.

[GATE 1994: 1 Mark]

Soln. $(Bandwidth)_{PCM} = nf_m$

Where n – number of bits in PCM code

f_m – signal bandwidth

$$n = log_2 L$$

$$n_1 = log_2 4 = 2$$

$$n_2 = log_2 64 = 6$$

$$(BW)_1 = n_1 f_m = 2f_m$$

$$(BW)_2 = n_2 f_m = 6f_m$$

$$\frac{(BW)_2}{(BW)_1} = \frac{6f_m}{2f_m} = 3\ times$$

$$(BW)_2 = 3(BW)_1$$

So, increase is 3 times

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22. The number of bits in a binary PCM system is increased from n to n+1. As a result, the signal quantization noise ratio will improve by a factor

(a) $(n+1)/n$

(b) $2^{(n+1)/n}$

(c) $2^{2(n+1)/n}$

(d) Which is independent of n

[GATE 1996: 2 Marks]

Soln. $\boldsymbol{SQNR = \frac{3}{2}2^{2n}}$

For $\boldsymbol{n_1 = n}$

$\boldsymbol{n_2 = n+1}$

$$(SQNR)_1 = \frac{3}{2}2^{2n}$$

$$(SQNR)_2 = \frac{3}{2}2^{2(n+1)} = \frac{3}{2}2^{2n+2} = \frac{3}{2}[2^{2n}.2^2]$$

$$\frac{(SQNR)_2}{(SQNR)_1} = \frac{2^2}{1} = 4$$

So, $(SQNR)_2 = 4(SQNR)_1$

Note that signal to quantization noise increases by factor of 4. So this is improvement in SQNR is independent of n

Option (d)

23. In a PCM system with uniform quantization, increasing the number of bits from 8 to 9 will reduce the quantization noise power by a factor of

(a) 9

(b) 8

(c) 4

(d) 2

[GATE 1998: 1 Mark]

Soln. Quantization noise in PCM is given by

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$$N_q = \frac{\Delta^2}{12}$$

$$stepsize\ (\Delta) = \frac{voltage\ range}{2^n} = \frac{V_{p-p}}{2^n}$$

$$N_q = \frac{V_{p-b}^2}{12 \times 2^{2n}} \quad so, N_q \propto \frac{1}{2^{2n}}$$

$$\frac{(N_q)_2}{(N_q)_1} = \frac{2^{2n_1}}{2^{2n_2}} = \frac{2^{2\times 8}}{2^{2\times 9}} = \frac{2^{16}}{2^{18}} = \frac{1}{2^2} = \frac{1}{4}$$

$$or \quad (N_q)_2 = \frac{(N_q)_1}{4}$$

Quantization noise reduces by a factor of 4

Option (c)

24. The peak to peak input to an 8 bit PCM coder is 2 volts. The signal power to quantization noise power ratio (in dB) for an input of $0.5 \cos \omega_m t$ is

(a) 47.8
(b) 43.8
(c) 95.6
(d) 99.6

[GATE 1999: 2 Marks]

Soln. Given

$$V_{p-p} = 2\ volts$$

No. of bits = 8

$$n = log_2 L$$

Or $8 = log_2 L \ \ or \ \ L = 2^7 = 128\ levels$

$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6n$$

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$$= 1.76 + 6 \times 7 = 43.8\ dB$$

Option (b)

25. A signal is sampled at 8 KHz and is quantized using 8-bit uniform quantizer. Assuming SNR_q for a sinusoidal signal, the correct statement for PCM signal with a bit rate of R is
(a) $R = 32\ kbps, SNR_q = 25.8\ dB$
(b) $R = 64\ kbps, SNR_q = 49.8\ dB$
(c) $R = 64\ kbps, SNR_q = 55.8\ dB$
(d) $R = 32\ kbps, SNR_q = 49.8\ dB$

[GATE 2003: 2 Marks]

Soln. **Given sampling rate = 8 KHz**

Then bit rate = $nf_s = 8 \times 8kHz = 64\ kbps$

$$SNR_q = (1.76 + 6.02\ .n)dB$$

$$= 1.76 + 6.02 \times 8$$

$$= 49.8\ dB$$

Option (b)

26. In a PCM system, if the code word length is increased from 6 to 8 bits, the signal to quantization noise ratio improves by the factor
(a) 8/6
(b) 12
(c) 16
(d) 8

[GATE 2004: 1 Mark]

Soln. Note that

$$\left(\frac{S}{N_q}\right)_{n=6} = 2^{2\times 6} = 2^{12}$$

$$\left(\frac{S}{N_q}\right)_{n=8} = 2^{2\times 8} = 2^{16}$$

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$$\frac{\left(\frac{S}{N_q}\right)_{n=8}}{\left(\frac{S}{N_q}\right)_{n=6}} = \frac{2^{16}}{2^{12}} = 2^4 = 16$$

Option (c)

27. A sinusoidal signal with peak to peak amplitude of 1.536 V is quantized into 128 levels using a midrise uniform quantizer. The quantization noise power is

(a) 0.768 V
(b) $48 \times 10^{-6} V^2$
(c) $12 \times 16^{-6} V^2$
(d) 3.072 V

[GATE 2003: 2 Marks]

Soln. Note

$$stepsize(\Delta) = \frac{V_{p-p}}{No. of\ levels} = \frac{1.536}{128} = 0.012V$$

Quantization noise

$$(N_q) = \frac{\Delta^2}{12} = \frac{(0.012)^2}{12} = 12 \times 10^{-6} V^2$$

Option (c)

28. A signal having uniformly distributed amplitude in the interval (-V to +V), is to be encoded using PCM with uniform quantization. The signal to quantizing noise ratio is determined by the

(a) dynamic
(b) sampling
(c) number of quantizing levels
(d) power spectrum of signal

[GATE 1988: 2 Marks]

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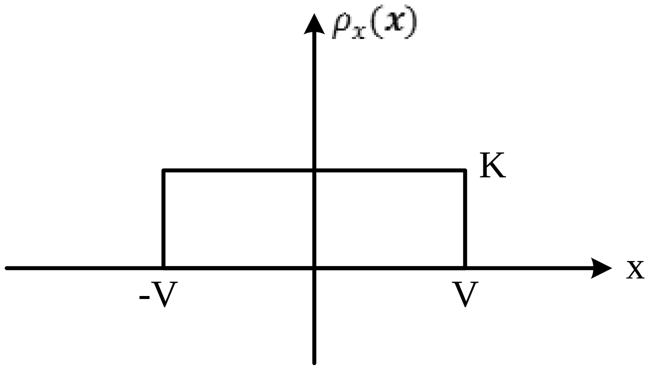
Soln. **The signal is uniformly distributed in the interval –V to +V, the PDF is shown in figure.**



Area under PDF is unity

$$K[V-(-V)] = 1 \quad or \quad 2VK = 1 \quad or \quad K = \frac{1}{2V}$$

So $\rho_x(x) = \begin{cases} \frac{1}{2V} & -V\ to\ V \\ 0 & otherwise \end{cases}$

Signal power

$$= s = \int_{-\infty}^{\infty} x^2\, \rho_x(x) dx = \int_{-V}^{V} x^2 . \frac{1}{2V} . dx = \frac{1}{2V}\left[\frac{x^3}{3}\right]_{-V}^{V} = \frac{1}{2V}\left[\frac{V^3}{3} - \frac{(-V^3)}{3}\right]$$

$$S = \frac{1}{2V} . \frac{2V^3}{3} = \frac{V^2}{3}$$

Quantization noise $(N_q) = \frac{\Delta^2}{12}$

Where

$$\Delta = \frac{V_{p-p}}{L} = \frac{V_{p-p}}{2^n}$$

$$N_q = \left(\frac{2V}{2^n}\right)^2 . \frac{1}{12} = \frac{4V^2}{12 \times 2^{2n}} = \frac{V^2}{3 \times 2^n}$$

$$\left(\frac{S}{N}\right)_q = \frac{V^2/3}{V^2/3 \times 2^n} = 2^n$$

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So,

$$\left(\frac{S}{N}\right)_q \propto 2^{2n}$$

So, signal to quantizing noise ratio is determined by number of quantizing levels

Option (c)

29. In a PCM systems, the signal $m(t) = \{\sin(100\,\pi t + \cos(100\,\pi t))\}$ is sampled at the Nyquist rate. The samples are processed by a uniform quantizer with step size 0.75V. The minimum data rate of the PCM system in bits per seccond is

[GATE 2014: 2 Marks]

Soln. Given, $m(t) = \sin 100\pi t + \cos 100\pi t = \sqrt{2}.\cos[100\pi t + \emptyset]$

Step size $\Delta = 0.75V$

$$\Delta = \frac{V_{p-p}}{L} = \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

Or,

$$L = \frac{2\sqrt{2}}{0.75} \cong 4 \quad so, \quad n = 2$$

Frequency of signal f = 50 Hz

Nyquist rate = 100

Bit rate $(R_b) = 2fs = 2 \times 100$

$$= 200\ bit/sec$$

$$= 200\ bps$$

30. An analog signal is band-limited to 4 KHz, sampled at the Nyquist rate and the samples levels are assumed to be independent and equally probable. If we transmit two quantized samples per second, the information rate is

(a) 1 bit/sec
(b) 2 bits/sec
(c) 3 bits/sec
(d) 4 bits/sec

[GATE 2011: 1 Mark]

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Soln. Signal is band limited to 4 KHz

Nyquist rate $= 2 \times f_m = 8KHz$

Levels = 4 i.e. $2^n = L \quad or \quad n = 2$

The number of bits = 2

Each sample requires 2 bits.

Two samples per second are transmitted so, the number of bits per second $\quad 2 \times 2 = 4\ bits/sec$

Option (d)

31. In a baseband communication link, frequencies up to 3500 Hz are used for, signaling. Using a raised cosine pulse with 75% excess bandwidth and for no inter-symbol interference, the maximum possible signaling rate in symbols per second is

(a) 1750 (c) 4000
(b) 2625 (d) 5250

[GATE 2012: 1 Mark]

Soln. Given

Signaling frequency (f) = 3500 Hz

Excess bandwidth used is $B = 0.75 \times 3500 = 2625\ Hz$

We know that

Bandwidth$\geq \frac{R_b}{2}$

Where R_b **is the data rate**

Minimum bandwidth required(B) $= \frac{R_b}{2}$

$$or, \qquad B = \frac{R_b}{2}$$

$$or,\ R_b = 2B = 2 \times 2625 = 5250\ Hz$$

Option (d)

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(c) Delta Modulation

32. In delta modulation, the slope overload distortion can be reduced by
    (a) decreasing the step size
    (b) decreasing the granular noise
    (c) decreasing the sampling noise
    (d) increasing the step size

[GATE 2007: 2 Marks]

Soln. When the slope of analog signal is much higher than that of approximated digital signal, then this difference is called slope overload distortion.

Condition to avoid slop overload in delta modulation is

$$\frac{\Delta}{Ts} \geq \frac{d}{dt}.m(t)$$

Where, $\Delta - \boldsymbol{Step\ size}$

$\boldsymbol{Ts - sampling\ interval}$

$\boldsymbol{m(t) - signal}$

From above equation we observe that if step size is increased, slope overload distortion can be avoided.

33. In the output of a DM speech encoder, the consecutive pulses are of opposite polarity during time interval $t_1 \leq t \leq t_2$. This indicates that during this interval
    (a) the input to the modulator is essentially constant
    (b) the modulator is going through slope overload
    (c) the accumulator is in saturation
    (d) the speech signal is being sampled at the Nyquist rate

[GATE 2004: 1 Mark]

Soln. Given

During the interval $\boldsymbol{t_1 \leq t \leq t_2}$

The consecutive pulses of encoder are of opposite polarity.

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In between the two adjacent sample values, if the baseband signal changes by an amount less than the step size, the output of Delta Modulator is sequence of alternate positive and negative pulses.

This small change in base band signal indicates that the baseband is almost constant

Option (a)

34. The input to a linear delta modulator having a step-size $\Delta = 0.628$ is a sine wave with frequency f_m and peak amplitude E_m. If the sampling frequency f_s = 40 KHz, the combination of the sine-wave frequency and the peak amplitude, where slope overload will take place is

	E_m	f_m
(a)	0.3 V	8 KHz
(b)	1.5 V	4 KHz
(c)	1.5 V	3 KHz
(d)	3.0 V	1 KHz

[GATE 2003: 2 Marks]

Soln. Given

$$\Delta = 0.628$$

$$m(t) = E_m \sin 2\pi f_m t$$

Slope overload takes place

When,

$$\frac{\Delta}{Ts} \leq \frac{d}{dt}.m(t)$$

$$or, \quad \frac{\Delta}{Ts} \leq 2\pi f_m.E_m$$

$$or, \quad \Delta.f_s \leq 2\pi f_m E_m$$

$$or, \quad \Delta.40 \times 10^3 \leq 2\pi f_m$$

$$or, \quad 0.628 \times 40 \times 10^3 \leq 6.28 f_m E_m$$

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$$or,\ \ 4 \times 10^3 \leq f_m E_m$$

(a) $0.3 \times 8KHz = 2.4KHz$
(b) $1.5 \times 4KHz = 6KHz$
(c) $1.5 \times 2KHz = 3KHz$
(d) $3 \times 1KHz = 3KHz$

Option (b)

35. The minimum step-size required for a Delta-Modulation operating at 32K samples/sec to track the signal (here u(t) is the unit-step function) $x(t) = 125t\{u(t) - u(t-1)\} + (250 - 125t)\{u(t-1) - u(t-2)\}$ so that slope-overload is avoided would be

(a) 2^{-10}
(b) 2^{-8}
(c) 2^{-6}
(d) 2^{-4}

[GATE 2006: 2 Marks]

Soln.

Given

$$m(t) = 125t[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

To avoid slope overload

$$\frac{\Delta}{Ts} \geq \frac{d}{dt} m(t)$$

$$\Delta \times 32 \times 1024 \geq 125$$

$$or,\ \ \Delta . 2^{15} \geq 125$$

$$or,\ \ \Delta \geq \frac{2^7}{2^{15}}$$

$$or,\ \ \Delta \geq 2^{-8}$$

Option (b)

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