Continuous Time Signals (Part - II) - Fourier Transform

1. The Fourier transform of a real valued time signal has
   (a) odd symmetry
   (b) even symmetry
   (c) conjugate symmetry
   (d) no symmetry

[GATE 1996: 1 Mark]

Soln. For real valued time signal, Fourier Transform has conjugate symmetry.

If $x(t)$ $is\ real$ $\rightarrow Fourier\ Transform$ is $X(f)$

Then there exists conjugate even symmetry (Also called Hermitian Symmetry)

i.e. $X(f) = X^*(-f)$

or $X^*(f) = X(-f)$

From above condition it can be shown that

$|X(f)|\ and\ R_e\{X(f)\}$ have even symmetry

i.e. $|X(f)| = |X(-f)|$

$\angle X(f) and\ I_m\{X(f)\}$ have odd symmetry

$\angle X(f) = -\angle X(-f)$

Option (c)

2. A signal x(t) has a Fourier transform X(ω). If x(t) is a real and odd function of t, them X(ω) is
   (a) a real and even function of ω
   (b) an imaginary and odd function of ω
   (c) an imaginary and even function of ω
   (d) a real and odd function of ω

[GATE 1999: 1 Mark]

Soln. If $f(t)$ is real and even then $F(\omega)$ is real

Even → $f(t) = f(-t)$

$F(\omega) = F(-\omega)$

Real → $f(-\omega) = f^*(\omega)$

Or $F(\omega) = F^*(\omega)$

If $f(t)$ is real and odd

$F(\omega)$ is pure imaginary

odd $\rightarrow f(t) = -f(-t)$

$$F(\omega) = -F(-\omega)$$

Option (b)

3. The Fourier transform of a conjugate symmetric function is always
(a) imaginary
(b) conjugate anti-symmetric
(c) real
(d) conjugate symmetric

[GATE 2004: 1 Mark]

Soln. Given that the time function $x(t)$ is conjugate symmetric i.e.

$$\text{If} \quad x(t) = x^*(-t)$$

Use the property of conjugate symmetry of FT

$$\text{If} \quad x(t) \rightarrow X(f)$$

Then $\quad x^*(-t) = X^*(f)$

Given $\quad x(t) = x^*(-t)$

Then $\quad X(f) = X^*(f)$

So, $\quad X(f)$ is real

Option (c)

4. If $G(f)$ represents the Fourier Transform of a signal $g(t)$ which is real and odd symmetric in time, then
(a) G(f) is complex
(b) G(f) is imaginary
(c) G(f) is real
(d) G(f) is real

[GATE 1992: 2 Marks]

Soln. $\quad g(t) \rightarrow G(f)$

Note, If $g(t)$ is real and even,

$G(f)$ is also real and even

But if $g(t)$ is real and odd

$G(f)$ is imaginary and odd

Option (b)

5. The amplitude spectrum of a Gaussian pulse is
   (a) uniform
   (b) a sine function
   (c) Gaussian
   (d) An impulse function

[GATE 1998: 1 Mark]

Soln. Gaussian pulse is defined by

$$f(t) = e^{-\pi t^2}$$

Fourier Transform of this pulse can be evaluated

$$\mathcal{F}\left[e^{-\pi t^2}\right] = \int_{-\infty}^{\infty} e^{-\pi t^2}\ e^{-j\omega t}\, dt$$

After evaluation of integral one gets

$$\mathcal{F}\left[e^{-\pi t^2}\right] = e^{-\pi f^2}$$

When area under Gaussian pulse and central ordinate of the pulse is unity, it is said to be normalized Gaussian pulse. Such pulse is its own Fourier Transform

$$e^{-\pi t^2} \longleftrightarrow e^{-\pi f^2}$$

Option (c)

6. The Fourier Transform of the signal $x(t) = e^{-3t^2}$ is of the following from where A and B are constants:
   (a) $A\ e^{-B|f|}$
   (b) $A\ e^{-Bf}$
   (c) $A + B|f|^2$
   (d) $A\ e^{-Bf^2}$

[GATE 2000: 1 Mark]

Soln. The Fourier Transform of a normalized Gaussian pulse is also normalized Gaussian pulse

For $g(t) = e^{-at^2}$

$$G(\omega) = \sqrt{\frac{\pi}{a}}\ e^{-\omega^2/4a}$$

So, it is of the form

$$Ae^{-Bf^2}$$

The constants A and B can be found

For $x(t) = e^{-3t^2}$

Here a = 3

So, $X(\omega) = \sqrt{\frac{\pi}{3}}\ .e^{-\omega^2/4\times 3}$

$$X(\omega) = \sqrt{\frac{\pi}{3}}.\ e^{-\omega^2/12}$$

Option (d)

7. The function f(t) has Fourier Transform g(ω). The Fourier Transform of

$$g(t) = \left(\int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt\right)\ is$$

(a) $\frac{1}{2\pi}f(\omega)$

(b) $\frac{1}{2\pi}f(-\omega)$

(c) $2\pi f(-\omega)$

(d) $none\ of\ above$

[GATE 1997: 1 Mark]

Soln. Given

$$f(t) \longleftrightarrow g(\omega)$$

Then F [g(t)] ?

Inverse transform

$$f(t) = \frac{1}{2}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$Or, \ \ 2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega) \ e^{j\omega t} \, d\omega$$

Here ω is dummy variable so can be exchanged

$$i.e \ \ 2\pi f(-\omega) = \int_{-\infty}^{\infty} F(t) e^{-j\omega t} \, dt = \mathcal{F}[f(t)]$$

Above equation shows that Fourier transform of time function f(t) is $2\pi f(-\omega)$

In this problem also, $g(\omega)$ is Fourier Transform for $f(t)$

So changing dummy variable (from t to ω) then $F\{g(t)\} = 2\pi f(-\omega)$

Option (c)

8. The Fourier transform of a function x(t) is X(f). The Fourier transform of $\frac{dx(t)}{dt}$ will be

(a) $\frac{dX(f)}{dt}$

(b) $j2\pi f \ X(f)$

(c) $jf \ X(f)$

(d) $\frac{X(f)}{jf}$

[GATE 1998: 1 Mark]

Soln.

$$If \ x(t) \longleftrightarrow X(f)$$

$$then \ \ \frac{dx}{dt} \longleftrightarrow j\omega \, X(\omega)$$

$$Since, \ \ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega$$

Then

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} . \frac{d}{dt} \left[\int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega \right]$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt}\{X(\omega)e^{j\omega t}\}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega\, X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \mathcal{F}^{-1}[j\omega\, X(\omega)]$$

$$= \mathcal{F}^{-1}[j2\pi\;\; X(f)]$$

This shows that differentiation in time domain is equivalent to multiplication by $j\omega = j2\pi f$ in frequency domain

Option (b)

9. The Fourier transform of a voltage signal x(t) is X(f). The unit of $|X(f)|$is

(a) Volt
(b) Volt – sec
(c) Volt / sec
(d) Volt^2

[GATE 1998: 1 Mark]

Soln. As per the definition of Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}.\, dt$$

Looking at R.H.S expression, then unit of X(f) will be volt – sec

Option (b)

10. If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

(a) E
(b) E/2
(c) 2E
(d) 4E

[GATE 2001: 1 Mark]

Soln. Given,

Signal $f(t)$ has energy E.

Find energy of the signal $f(2t)$.

Energy of signal

$$f(t) = \int_{-\infty}^{\infty} f^2(t)dt$$

So, energy of signal $f(2t)$ will be

$$= \int_{-\infty}^{\infty} f^2(2t)\ dt$$

$$= \int_{-\infty}^{\infty} f^2(\tau)\ \frac{d\tau}{2} = \frac{E}{2}$$

Option (b)

11.The Fourier transform $F\{e^{-t}\ u(t)\}$ is equal to

$$\frac{1}{1+j2\pi f}\ Therefore,\quad F\left\{\frac{1}{1+j2\pi t}\right\}\ is$$

(a) $e^{f}u(f)$

(b) $e^{-f}u(f)$

(c) $e^{f}u(-f)$

(d) $e^{-f}u(-f)$

[GATE 2002: 1 Mark]

Soln. Given,

$$\mathcal{F}[e^{-t}\ u(t)] = \frac{1}{(1+j2\pi f)}$$

Using the duality property

If $\quad g(t) \rightarrow G(f)$

$\quad G(t) \rightarrow g(-f)$

Therefore,

$$\frac{1}{(1+j2\pi t)} \rightarrow e^{f}u(-f)$$

Option (c)

12.Let $x(t) \leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(a) $\frac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(b) $\frac{1}{5}e^{\frac{j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$

(c) $\frac{1}{5}e^{-j3\omega}X\left(\frac{j\omega}{5}\right)$

(d) $\frac{1}{5}e^{j3\omega}X\left(\frac{j\omega}{5}\right)$

[GATE 2006: 1 Mark]

Soln. Given,

$$x(t) \longleftrightarrow X(j\omega)$$

Find Fourier Transform of $x(5t-3)$

Time shifting property

$$x(t \mp t_0) \longrightarrow e^{\pm jt_0\omega} X(j\omega)$$

Scaling property

$$x(Kt) \longrightarrow \frac{1}{|K|} X\left(j\frac{\omega}{K}\right)$$

Using time shifting property

$$x(t-3) \longrightarrow e^{-j3\omega} . X(j\omega)$$

Using scaling property

$$x(5t-3) = \frac{1}{5}e^{-j\frac{3\omega}{5}} \times \left(\frac{j\omega}{5}\right)$$

Option (a)

13. If the Fourier Transform of a deterministic signal g(t) is G(f), then

Items – 1

(1) The Fourier Transform of $g(t-2)$ is

(2) The Fourier Transform of $g(t/2)$ is

Items – 2

(A) $G(f)e^{-j\,(4\pi f)}$

(B) G(2f)

(C) 2G(2f)

(D) G(f - 2)

Match each of the items 1, 2 on the left with the most appropriate item A, B, C, or D on the right.

[GATE 1997: 2 Marks]

Soln. $g(t) \;\leftrightarrow\; G(f)$

$$g(t-2) \;\leftrightarrow\; e^{-j2\pi 2f}\, G(f) = G(f)e^{-j(4\pi f)}$$

$$g\left(\frac{t}{2}\right) \;\leftrightarrow\; \left(\frac{1}{1/2}\right) G\left(\frac{f}{1/2}\right) = 2G(2f)$$

Option 1 – A, 2 – C

14. Let x(t) and y(t) (with Fourier transform X(f) and Y(f) respectively) be related as shown in Figure (1) & (2).

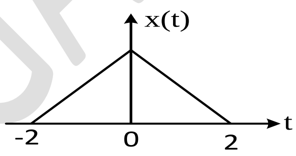
Then $Y(f)$ is

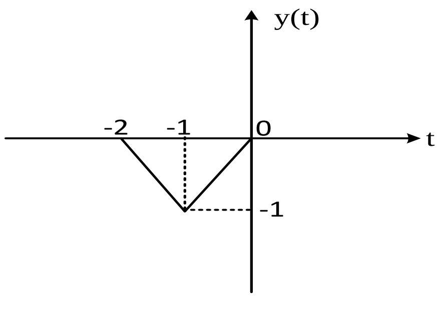
(a) $-\frac{1}{2}X(f/2)e^{-j2\pi f}$

(b) $-\frac{1}{2}X(f/2)e^{j2\pi f}$

(c) $-X(f/2)e^{j2\pi f}$

(d) $-X(f/2)e^{-j2\pi f}$





[GATE 2004: 2 Marks]

Soln. The figures of $x(t)$ and $y(t)$ are given, from these figures.

$$y(t) = -x(2t+2)$$

$$If\ x(t) \leftrightarrow X(f)$$

$$Then\ x(t+2) \rightarrow e^{j2\pi 2f} X(f)$$

Using time shifting property

$$x(2t+2) \rightarrow \left(\frac{1}{2}\right) X(f/2) e^{j2\pi f}$$

According to time scaling property

$$y(f) = -\frac{1}{2} X(f/2) e^{j2\pi f}$$

Option (b)

15. For a signal x(t) the Fourier transform is X(f). Then the inverse Fourier transform of X(3f + 2) is given by

(a) $\frac{1}{2} x\left(\frac{t}{2}\right) e^{j3\pi t}$

(b) $\frac{1}{3} x\left(\frac{t}{3}\right) e^{-j4\pi/3}$

(c) $3x(3t)e^{-j\,4\pi t}$

(d) $x(3t+2)$

[GATE 2005: 2 Marks]

Soln. In this problem we use the following two properties of Fourier Transform

If $x(t) \rightarrow X(f)$

$$e^{\mp j2\pi f_0 t} x(t) \longrightarrow X(f \pm f_0) \quad ----(1)$$

Frequency shifting property

$$\frac{1}{|k|} x\left(\frac{t}{k}\right) \longrightarrow (kf) \quad ----------(2)$$

Time scaling property

Using frequency shift property

$$e^{-j4\pi t} x(t) \longrightarrow X(f+2)$$

Using time scaling property

$$\frac{1}{3}x\left(\frac{t}{3}\right)e^{-4\pi t/3} \longrightarrow X(3f+2)$$

Option (b)

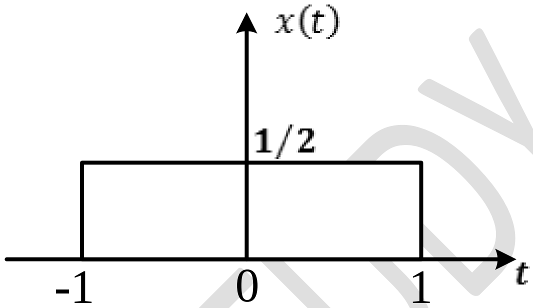
16. Two of the angular frequencies at which its Fourier transform becomes zero are

(a) $\pi, 2\pi$ (c) $0, \pi$

(b) $0.5\,\pi, 1.5\,\pi$ (d) $2\,\pi, 2.5\,\pi$

[GATE 2008: 2 Marks]

Soln. The given time function $x(t)$ is shown is figure



Its Fourier Transform $X(f)$ is given by

$X(f) = 2\sin c(2f)$

$= 2\ for\ f = 0$

$= 0\ for\ 2f = \pm 1,\ \pm 2, - - - -$

Or $\omega = 2\pi f = \pm\pi,\ \pm 2\pi,\ \pm 3\pi, - - - - -$

Option (a)

17. The Fourier transform of a signal h(t) is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of h(0) is

(a) 1/4 (c) 1

(b) 1/2 (d) 2

[GATE 2012: 2 Marks]

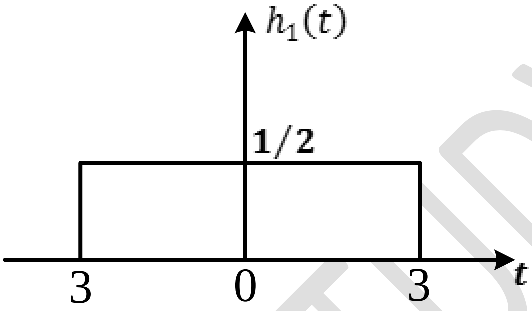
Soln.

$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega}$$

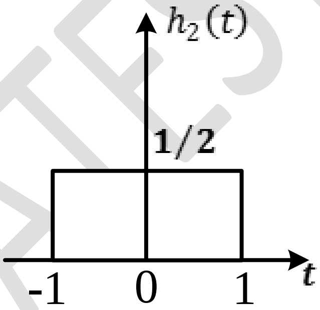
$$= \frac{2\sin 2\omega \ . \ \cos\omega}{\omega}$$

$$= \frac{\sin 3\omega + \sin\omega}{\omega}$$

$$= \frac{\sin 3\omega}{\omega} + \frac{\sin\omega}{\omega}$$



$$\longleftrightarrow \quad \frac{\sin 3\omega}{\omega}$$



$$\longleftrightarrow \quad \frac{\sin\omega}{\omega}$$

So, inverse Fourier Transform of $H(j\omega)$

$h(t) = h_1(t) + h_2(t)$

$$h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1$$

Option (c)

18. Let $g(t) = e^{-\pi t^2}$, and $h(t)$ is filter marched to $g(t)$. If $g(t)$ is applied as input to $h(t)$, then the Fourier transform of the output is

(a) $e^{-\pi t^2}$

(b) $e^{-\pi f^2/2}$

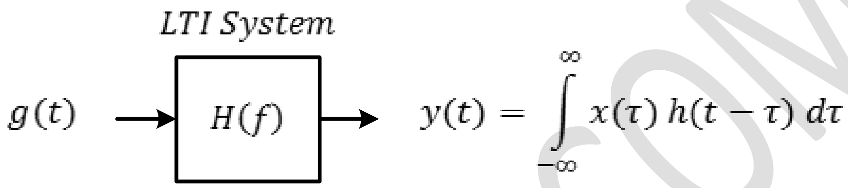
(c) $e^{-\pi |f|}$

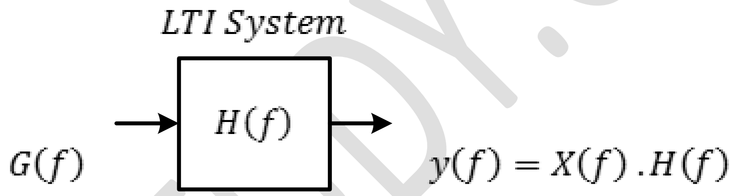
(d) $e^{-2\pi f^2}$

[GATE 2013: 1 Mark]

Soln. Given, $g(t) = e^{-\pi t^2}$

$h(t)$ *is matched to* $g(t)$





$g(t) = e^{-\pi t^2}$ (*Gaussian Pulse*)

$G(f) = e^{-\pi f^2}$ (*Fourier Transform of Gaussian Pulse*)

$h(f) = e^{-\pi f^2}$ (*Since filter is matched*)

$y(f) = G(f)\,.h(f) = e^{-\pi f^2}\,.\,e^{-\pi f^2}$

$y(f) = e^{-2\pi f^2}$

Option (d)

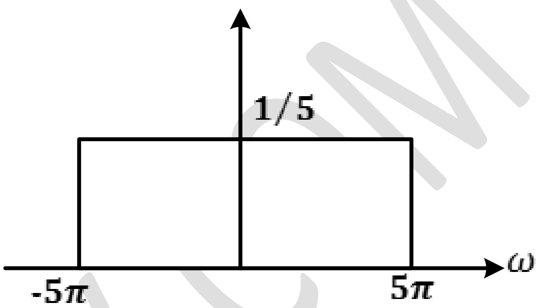
19. The value of the integral

$$\int_{-\infty}^{\infty} \sin c^2(dt) \quad is \text{ \_\_\_\_\_\_\_\_ }.$$

[GATE 2014: 1 Mark]

Soln. The given integral gives the energy of the signal $\sin c\ (5t)$

$$\sin c\ (5t) = \frac{\sin 5\pi t}{5\pi t} \longleftrightarrow$$



Using Perceval's theorem

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2\ dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2\ d\omega$$

$$Energy = \frac{1}{2\pi} \int_{-5\pi}^{5\pi} (1/5)^2\ d\omega$$

$$= \frac{1}{50\pi}(10\pi) = \frac{1}{5} = 0.2$$

Answer 0.2