Time Response Analysis (Part – II)

1. A critically damped, continuous-time, second order system, when sampled, will have (in Z domain)
   (a) A simple pole
   (b) Double pole on real axis
   (c) Double pole on imaginary axis
   (d) A pair of complex conjugate poles

[GATE 1988: 2 Marks]

Soln. For critically damped continuous time second order system roots of denominator are: $s^2 + 2\,\xi\omega_n s + \omega_n^2 = 0 \quad when \;\; \xi = 1, s^2 + 2\omega_n s + \omega_n^2 = 0$

$$(s + \omega_n)^2 = 0$$

Root are $s = -\omega_n, -\omega_n$

Double pole on real axis

Option (b)

2. A second-order system has transfer function given by $G(s) = \frac{25}{S^2+8s+25}$. If the system, initially at rest, is subjected to a unit step input at t = 0, the second peak in the response will occur at
   (a) $\pi\ sec$
   (b) $\pi/3\ sec$
   (c) $2\pi/3\ sec$
   (d) $\pi/2\ sec$

[GATE 1991: 2 Marks]

Soln. $G(s) = \frac{25}{S^2+8s+25}$

$$\omega_{n^2} = 25$$

$$\omega_n = 5$$

$$2\,\xi\omega_n = 8$$

$$\xi = \frac{8}{10} = 0.8$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5\sqrt{1-(0.8)^2} = 3$$

for 2nd peak n = 3

$$t_p = \frac{n\pi}{\omega_d} = \frac{3\pi}{3} = \pi\ sec$$

Option (a)

3. The poles of a continuous time oscillator are ……..

[GATE 1994: 1 Mark]

Soln. The poles of a continuous time oscillator are imaginary

4. The response of an LCR circuit to a step input is
   (a) Over damped
   (b) Critically damped
   (c) Oscillatory
   If the transfer function has
   (i) poles on the negative real axis
   (ii) poles on the imaginary axis
   (iii) multiple poles on the positive real axis
   (iv) poles on the positive real axis
   (v) multiple poles on the negative real axis

[GATE: 1994: 2 Marks]

Soln. The response of an LCR circuit to a step input

(a) over damped case $\xi > (1)$ *poles on the negative real axis*

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2}$$

Roots of denominator are $s = -\xi\,\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

(b) critically damped $\xi = 1$ – – – – – (5)

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\,\omega_n s + \omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\omega_n)^2}, \quad s = -\omega_n, -\omega_n$$

It has multiple poles on the negative real axis

(c) oscillatory – (2) poles on the imaginary axis

$\xi = 0$

$s = \pm\ j\omega_n$

Option a –1, b – 5, c – 2

5. Match the following codes with List-I with List-II:

List – I

(a) Very low response at very high frequencies

(b) Over shoot

(c) Synchro-control transformer output

List – II

(i) Low pass systems

(ii) Velocity damping

(iii) Natural frequency

(iv) Phase-sensitive modulation

(v) Damping ratio

[GATE 1944: 2 Marks]

Soln. (a) very low response at very high frequencies → low pass system

(b) over sheet → Damping ratio

(c) synchro – control transformer output → phase sensitive modulation

Option: a – 1, b – 5, c – 4

6. For a second order system, damping ratio $(\xi), is\ 0 < \xi < 1$, then the roots of the characteristic polynomial are

(a) real but not equal
(b) real and equal
(c) complex conjugates
(d) imaginary

[GATE 1995: 1 Mark]

Soln. $\boldsymbol{\xi} < \mathbf{1}$ **underdamped system, roots of denominator**

$$\boldsymbol{s^2 + 2\,\xi\,\omega_n s + \omega_n^2 = 0}$$

Are $\boldsymbol{s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$

Roots are complex conjugate

Option (c)

7. If $L[f(t)] = \frac{2(S+1)}{S^2+2S+5}\ then\ f(0^+)\ and\ f(\infty)$ are given by

(a) 0,2 respectively
(b) 2. 0 respectively
(c) 0, 1 respectively
(d) 2/5, 0 respectively

[Note: L' stands for 'Laplace Transform of]

[GATE 1995: 1 Mark]

Soln. $\boldsymbol{F(s) = \frac{2(s+1)}{s^2+2s+5}}$

$$\boldsymbol{f(0^+) = \lim_{s\to\infty} SF(s) = \frac{2s^2+2s}{s^2+2s+5}}$$

$$\boldsymbol{= \lim_{s\to\infty} \frac{2+\frac{2}{s}}{1+\frac{2}{s}+\frac{5}{s^2}} = \frac{2+0}{1+0} = 2}$$

$$\boldsymbol{f(\infty) = \lim_{s\to 0} SF(s) = \frac{2s^2+2s}{s^2+2s+5} = 0}$$

Option (b)

8. The final value theorem is used to find the
   (a) steady state value of the system output
   (b) initial value of the system output
   (c) transient behavior of the system output
   (d) none of these

[GATE 1995: 1 Mark]

Soln. The final value theorem is used to find the steady state value of the system.

Option (a)

9. $If\ F(s) = \frac{\omega}{S^2+\omega^2}$, then the value of $\lim_{t\to\infty} f(t)$,
   $\{where\ F(s) is\ the\ L[f(t)]\}$
   (a) cannot be determined
   (b) is zero
   (c) is unity
   (d) is infinite

[GATE 1998: 1 Mark]

Soln. $\boldsymbol{F(s) = \frac{\omega}{s^2+\omega^2}}$

$\boldsymbol{F(s) = \frac{\omega}{s^2+\omega^2}}$ **has poles $\boldsymbol{s \pm j\omega}$ (pure imaginary) it is oscillatory function hence final value $\boldsymbol{\lim_{t\to\infty} f(t)}$ can not be determined**

10. Consider a feedback control system with loop transfer function

$$G(s)H(s) = \frac{K(1+0.5s)}{s(1+s)(1+2s)}$$

The type of the closed loop system is
   (a) zero
   (b) one
   (c) two
   (d) three

[GATE 1998: 1 Mark]

Soln. $$\boldsymbol{G(s)\ H(s) = \frac{k(1+0.5s)}{s(1+s)(1+2s)}}$$

The number of poles at the origin of open loop transfer function given the type of the system. It is a type one system

Option (b)

11. The unit impulse response of a linear time invariant system is the unit step function u(t). For t > 0, the response of the system to an excitation $e^{-at}u(t), a > 0$ will be

(a) ae^{-at}

(b) $(1/a)(1 - e^{-at})$

(c) $a(1 - e^{-at})$

(d) $1 - e^{-at}$

[GATE 1998: 2 Marks]

Soln. $\boldsymbol{H(s) = \frac{1}{s}}$

System excitation $\boldsymbol{r(t) = e^{-at}\, u(t)}$

$$\boldsymbol{R(s) = \frac{1}{s+a}}$$

Response of the system $\boldsymbol{C(s) = R(s)\, H(s) = \frac{1}{s(s+a)}}$

$$\boldsymbol{C(s) = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{(s+a)}\right]}$$

$$\boldsymbol{C(t) = \frac{1}{a}[1 - e^{-at}]}$$

Option (b)

12. If the characteristic equation of a closed-loop system is $s^2 + 2s + 2 = 0$, then the system is

(a) Overdamped

(b) Critically damped

(c) Under damped

(d) undamped

Soln. **Characteristic equation of closed loop system is $s^2 + 2s + 2 = 0$**

Comparing with 2nd order equation

$s^2 + 2\,\xi\omega_n + \omega_n^2 = 0$

$2\,\xi\omega_n = 2$

$\xi = \frac{1}{\omega_n}$

$\omega_n^2 = 2, \quad \omega_n = \sqrt{2}, \;\; \xi = \frac{1}{\sqrt{2}}$ **which is less than 1**

It is case of under damped

Option (c)

13. Consider a system with the transfer function $(s) = \frac{s+6}{Ks^2+s+6}$. Its damping ratio will be 0.5 when the value of K is

(a) 2/6
(b) 3
(c) 1/6
(d) 6

[GATE 2002: 1 Mark]

Soln. **Transfer function $G(s) = \frac{s+6}{ks^2+s+6}$**

Damping ratio $\xi = 0.5$

Comparing with 2nd order equation $s^2 + 2\,\xi\omega_n s + \omega_n^2$

$G(s) = \frac{s+6}{k\left[s^2+\frac{s}{k}+\frac{6}{k}\right]}$

${\omega_n}^2 = \frac{6}{k}, \quad \omega_n = \sqrt{\frac{6}{k}}$

$2\,\xi\omega_n = \frac{1}{k}$

$$2\ \xi\sqrt{\frac{6}{k}} = \frac{1}{k}$$

$$2 \times 0.5\sqrt{\frac{6}{k}} = \frac{1}{k}$$

$$or \quad \frac{6}{k} = \frac{1}{k^2}$$

$$or \quad k = \frac{1}{6}$$

Option (c)

14. A casual system having the transfer function $H(s) = \frac{1}{s+2}$ is excited with $10u(t)$. The time at which the output reaches 99% of its steady state value is

(a) 2.7 sec
(b) 2.5 sec
(c) 2.3 sec
(d) 2.1 sec

[GATE 2004: 2 Marks]

Soln. $$H(s) = \frac{1}{s+2}$$

$$r(t) = 10\mu(t)$$

$$R(s) = \frac{10}{s}$$

$$C(s) = H(s)\ R(s) = \frac{10}{s(s+2)}$$

$$C(s) = \frac{5}{s} - \frac{5}{s+2}$$

$$C(t) = 5[1 - e^{-2t}]$$

Steady state value = 5

99% of the steady value reaches at

$5[1-e^{-2t}] = \frac{5\times 99}{100}$

$5(1-e^{-2t}) = 5\times 0.99$

$1-e^{-2t} = 0.99$

Or $e^{-2t} = 0.1$

$-2t = l_n(0.1)$

$t = 2.3\ sec$

Option (c)

15. The transfer function of a plant is $(s) = \frac{5}{(s+5)(s^2+s+1)}$. The second order approximation of T(s) using dominant pole concept is

(a) $\frac{1}{(s+5)(s+1)}$

(b) $\frac{5}{(s+5)(s+1)}$

(c) $\frac{5}{(s^2+s+1)}$

(d) $\frac{1}{(s^2+s+1)}$

[GATE 2007: 2 Marks]

Soln. Transfer function $= \frac{5}{(s+5)(s^2+s+1)}$

$$= \frac{5}{5\left(\frac{s}{5}+1\right)(s^2+s+1)}$$

The poles nearer to imaginary axis dominates nature of time response and are called dominant poles. The factor that has to be eliminated should be in time constant form so, $T(s) = \frac{1}{s^2+s+1}$

Option (d)

16. Group I lists a set of four transfer functions.

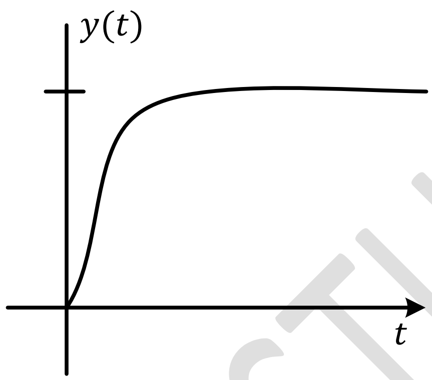
Group II gives a list of possible step responses y(t). Match the step responses with the corresponding transfer functions.

Group I

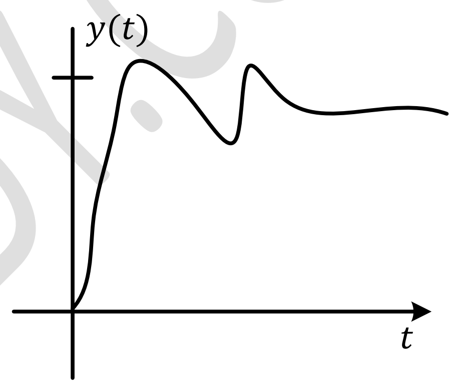
$P = \dfrac{25}{s^2 + 25}$ $\qquad$ $R = \dfrac{36}{s^2 + 12s + 36}$

$Q = \dfrac{36}{s^2 + 20s + 36}$ $\qquad$ $S = \dfrac{36}{s^2 + 7s + 49}$

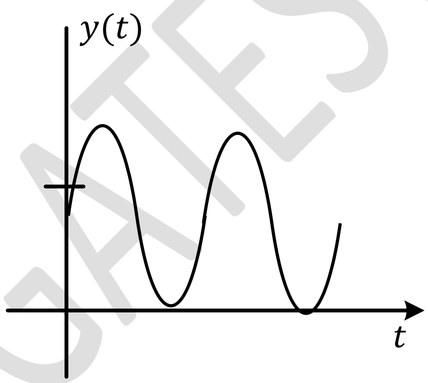
1.



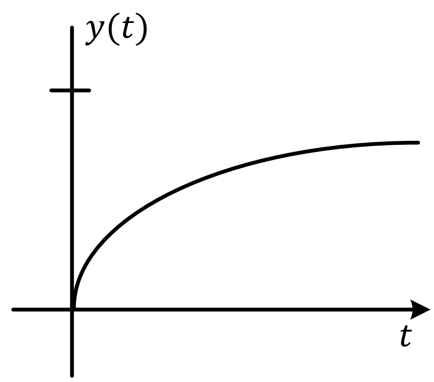
2.



3.



4.



(a) P – 3, Q – 1, R – 4, S – 2
(b) P – 3, Q – 2, R – 4, S – 1
(c) P – 2, Q – 1, R – 4, S – 3
(d) P – 3, Q – 4, R – 1, S – 2

[GATE 2008: 2 Marks]

Soln. **The figure 1 is a case of critically damped, Figure 2 underdamped, Figure 3 undamped, figure 4 is overdamped**

The given transfer function

$$P = \frac{25}{s^2 + 25} \qquad R = \frac{36}{s^2 + 12s + 36}$$

$$Q = \frac{36}{s^2 + 20s + 36} \qquad S = \frac{36}{s^2 + 7s + 49}$$

Comparing the given transfer function with

$$\frac{\omega_n^2}{s^2 + 2\,\xi\,\omega_n s + \omega_n^2}, \quad P = \frac{25}{s^2+25}$$

$$\omega_n = 5, \quad \xi = 0$$

P is undamped, matched to figure 3 so P – 3

$$Q = \frac{36}{s^2+20s+36} \quad , \quad \omega_n = 6$$

$$2\,\xi\,\omega_n = 20$$

$$\xi = \frac{20}{2\times 6} = 1.67$$

Q is overdamped matched to fig. 4 Q – 4

$$R = \frac{36}{s^2+12s+36} \quad , \quad \omega_n = 6$$

$$2\,\xi\,\omega_n = 12$$

$$\xi = \frac{12}{2\times 6}$$

$$= 1$$

R is critically damped matched to fig 1, R – 1

$$S = \frac{49}{s^2+7s+49} \quad , \quad \omega_n = 7$$

$$2\,\xi\,\omega_n = 7$$

$$\xi = \frac{7}{2\times7} = 0.5$$

= 1

S is underdamped matched to fig 2

Option (d) P – 3, Q – 4, R – 1, S – 2

17. The unit step response of an under-damped second order system has steady state value of –2. Which one of the following transfer functions has these properties?

(a) $\frac{-2.24}{s^2+2.59s+1.12}$

(c) $\frac{-2.24}{s^2-2.59s+1.12}$

(b) $\frac{-3.82}{s^2+1.91s+1.91}$

(d) $\frac{-3.82}{s^2-1.91s+1.91}$

[GATE 2009: 2 Marks]

Soln. Steady state value = - 2

The 2nd order $T/F = \frac{\omega_n^2}{s^2+2\,\xi\,\omega_n s+\omega_n^2}$

For underdamped system $\xi < 1$

Option (c) and (d) are wrong because the system is unstable as ξ **is negative**

In option (a) $\omega_n = \sqrt{1.12} = 1.06$

$$2\,\xi\,\omega_n = 2.59$$

$$\xi = \frac{2.59}{2\times1.06} = 1.22$$

$\xi > 1$ **hence it is wrong**

In option (b) $\quad T.F = \frac{-3.82}{s^2+1.91s+1.91}$

$$\omega_n = \sqrt{1.91} = 1.38$$

$$2\,\xi\,\omega_n = 1.91$$

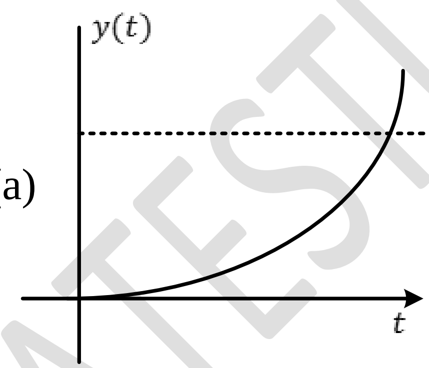
$$\xi = \frac{1.91}{2\times1.38} = 0.69$$

$\xi < 1$ **so system is underdamped**

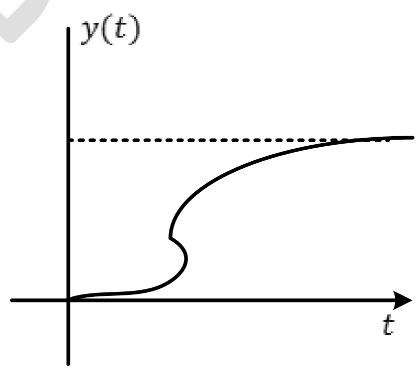
Option (b)

18. The differential equation $100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$ describes a system with an input $x(t)$ and an output $y(t)$. The system, which is initially relaxed, is exited, by a unit step input. The output y(t) can be represented by the waveform.

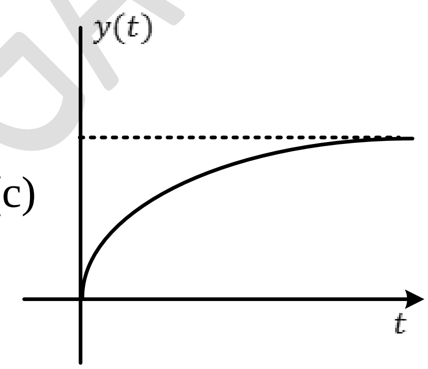
(a)



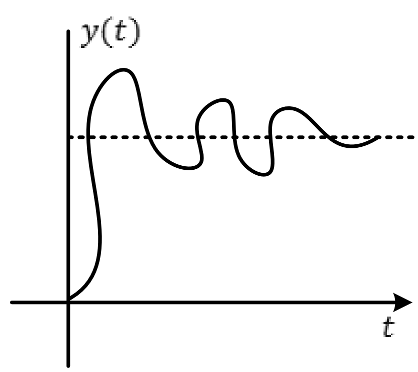
(b)



(c)



(d)



[GATE 2011: 1 Mark]

Soln. $$100\frac{d^2y}{dt^2} - 20\frac{dy}{dt} + y = x(t)$$

Taking Laplace transform of both sides

$$100\, s^2 y(s) - 20\, s\, y\,(s) + y(s) = X(s) = \frac{1}{s}$$

$$(100s^2 - 20s + 1)y(s) = \frac{1}{s}$$

$$y(s) = \frac{1}{s(10s-1)^2} \quad \textbf{Poles are at } s = 0, \frac{1}{10}, \frac{1}{10}$$

Poles are on the right hand side of s plane so given system is unstable

Option (a) represents unstable system.

19. The open-loop transfer function of a dc motor is given as

$$\frac{\omega(s)}{V_a(a)} = \frac{10}{1+10s}$$

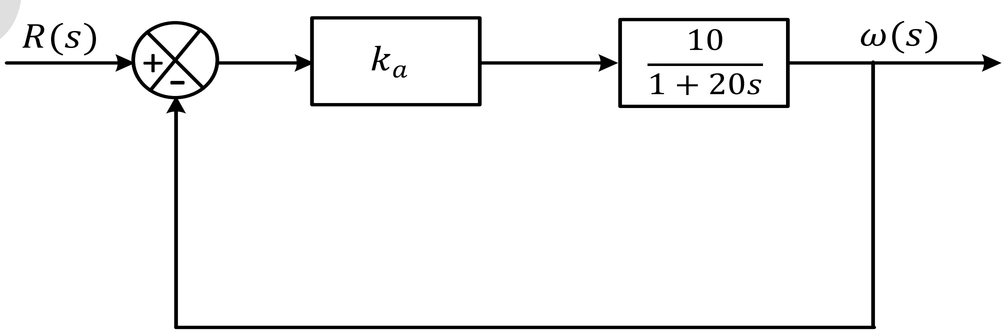
when connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is

(a) 1
(b) 5
(c) 10
(d) 100

[GATE 2013: 2 Marks]

Soln. The open loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$

When connected in feedback is shown below.



Open loop transfer function $= \frac{10\ k_a}{1+10\ s}$

$$G(s) = \frac{k_a}{s+\frac{1}{10}}$$

$$g(t) = e^{-\frac{t}{10}}$$

Loop time constant $\tau_{0l} = 10$

Closed loop transfer function $= H(s) = \frac{G(s)}{1+G(s)}$

$$H(s) = \frac{10k_a/1+10s}{1+(10k_a)/1+10s} = \frac{10k_a}{1+10s+10k_a}$$

$$H(s) = \frac{10k_a}{1+10s+10k_a} = \frac{k_a}{s+\left(k_a+\frac{1}{10}\right)}$$

$$h(t) = k_a e^{-\left(\frac{1}{k_a+0.1}\right)t}$$

Time constant of closed loop system $= \frac{1}{\left(k_a+\frac{1}{10}\right)}$

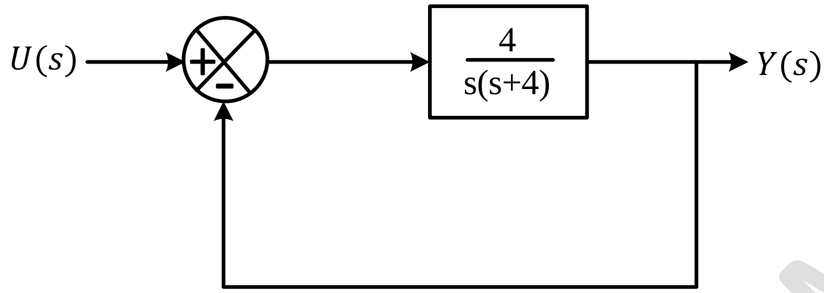
$$\tau_{cl} = \frac{\tau_{ol}}{100} = \frac{1}{k_a+0.1}$$

$$= \frac{10}{100} = \frac{1}{k_a+0.1}$$

or $k_a = 10 - 0.1 = 9.9 \cong 10$

Option (c)

20. For the second order closed-loop system shown in the figure, the natural frequency (in rad/s) is



(a) 16
(b) 4
(c) 2
(d) 1

[GATE 2014: 1 Mark]

Soln. $$\frac{Y(s)}{U(s)} = \frac{4/s(s+4)}{1+4/s(s+4)}$$

$$= \frac{4}{s(s+4)+4}$$

$$= \frac{4}{s^2+4s+4}$$

Comparing with 2nd order system transfer

Function $\frac{\omega_n{}^2}{s^2+2\,\xi\,\omega_n s+\omega_n^2}$

$\omega_n^2 = 4, \quad or \quad \omega_n = 2$

Option (c)