

Frequency Response Analysis (Part - I)

1. A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot having a slope of:

(a) – 40 dB/decade

(c) – 280 dB/decade

(b) – 240 dB/decade

(d) – 320 dB/decade

[GATE 1987: 2 Marks]

Soln. Poles (P) = 14

$$\text{Zeros (z)} = 2$$

$$P - Z = 14 - 2$$

$$= 12$$

$$\lim_{\omega \rightarrow \infty} \text{slope} = (P - Z) \left(-\frac{20\text{dB}}{\text{dec}} \right)$$

$$= -240\text{dB/decade}$$

Option (b)

2. The polar plot of $G(s) = \frac{10}{s(s+1)^2}$ intercepts real axis at $\omega = \omega_0$. Then, the real part and ω_0 are respectively given by:

(a) – 2.5, 1

(c) – 5, 1

(b) – 5, 0.5

(d) – 5, 2

[GATE 1987: 2 Marks]

Soln. $G(s) = \frac{10}{s(s+1)^2} = \frac{10}{s(s+1)(s+1)}$

$$\angle G(j\omega) = -90^\circ - 2 \tan^{-1} \omega$$

ω_{pc} is the phase cross over frequency where

$$\angle G(j\omega) = -180^\circ$$

$$\text{so } -180^\circ = -90^\circ - 2 \tan^{-1} \omega_{pc}$$

$$2 \tan^{-1} \omega_{pc} = 90^\circ$$

$$\omega_{pc} = 1 \text{ rad/sec}$$

$$\begin{aligned}
 |G|_{\omega=\omega_{pc}} &= \frac{10}{\omega\sqrt{1+\omega^2}\sqrt{1+\omega^2}} \\
 &= \frac{10}{1 \times \sqrt{2} \times \sqrt{2}} \\
 &= \frac{10}{2} = 5
 \end{aligned}$$

At $\omega = \omega_{pc}$ the polar plot crosses the negative real axis at -5

Option (c)

3. From the Nicholas chart one can determine the following quantities pertaining to a closed loop system:

- (a) Magnitude and phase (c) Only magnitude
 (b) Band width (d) Only phase

[GATE 1989: 2 Marks]

Soln. Nicholas chart is magnitude versus phase plot

4. The open-loop transfer function of a feedback control system is

$$G(s).H(s) = \frac{1}{(s+1)^3}$$

The gain margin of the system is

- (a) 2 (c) 8
 (b) 4 (d) 16

[GATE 1991: 2 Marks]

Soln.

$$G(s).H(s) = \frac{1}{(s+1)^3}$$

$$GM = \frac{1}{|G(j\omega_{pc})H(j\omega_{pc})|} = \frac{1}{M}$$

ω_{pc} is the phase cross over frequency where

$$\angle G(s)H(s) = -180^\circ$$

$$G(s)H(s) = \frac{1}{(s+1)(s+1)(s+1)}$$

$$-3\tan^{-1}\omega_{pc} = -180^\circ$$

$$\tan^{-1}\omega_{pc} = -60^\circ$$

$$\omega_{pc} = \sqrt{3} \text{ rad/sec}$$

$$M = |G(j\omega_{pc}) H(j\omega_{pc})| = \frac{1}{(\sqrt{1+\omega_{pc}^2})^3}$$

$$= \frac{1}{8}$$

$$GM = \frac{1}{M} = 8$$

Option (c)

5. Non-minimum phase transfer function is defined as the transfer function
- (a) which has zero in the right-half s-plane
 - (b) which has zero only in the left-half s-plane
 - (c) which has poles in the right-half s-plane
 - (d) which has poles in the left-half s-plane

[GATE 1995: 1 Mark]

Soln. Non minimum phase transfer function is defined as the transfer function which has one or more zeros in the right half of s – plane and remaining poles and zeros in the left half of s – plane.

Option (a)

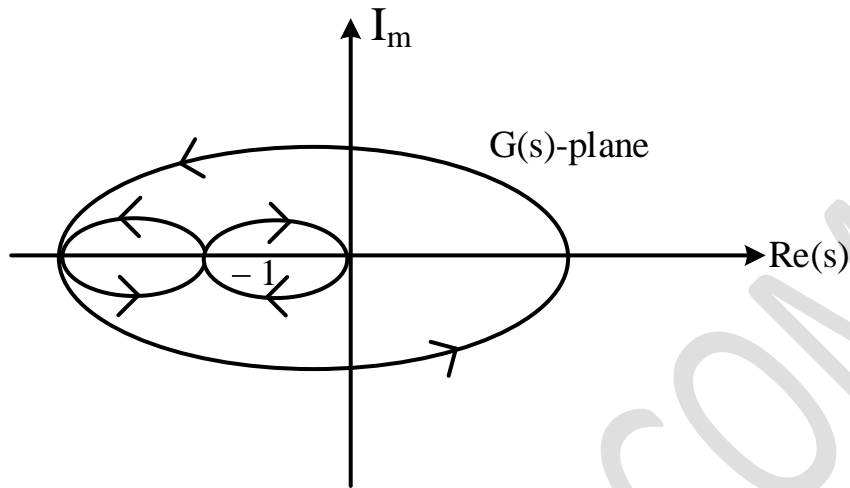
6. The Nyquist plot of a loop transfer function $G(j\omega) H(j\omega)$ of a system encloses the $(-1, j0)$ point. The gain margin of the system is
- (a) less than zero
 - (b) zero
 - (c) greater than zero
 - (d) infinity

[GATE 1998: 1 Mark]

Soln. A system is unstable when Nyquist plot of $G(j\omega) H(j\omega)$ enclosed the point $(-1, j0)$. Gain margin of unstable system is less than zero

Option (a)

7. The Nyquist plot for the open-loop transfer function $G(s)$ of a unity negative feedback system is shown in the figure, if $G(s)$ has no pole in the right-half of s -plane, the number of roots of the system characteristic equation in the right-half of s -plane is



- (a) 0
 (b) 1
 (c) 2
 (d) 3

[GATE 2001: 1 Mark]

Soln. $N = P - Z$

One encirclement in clockwise direction and one in anticlockwise direction hence $N = 0$

Given that number of poles of $G(s)H(s)$ in the right half s - plane, $p = 0$

$$N = P - Z$$

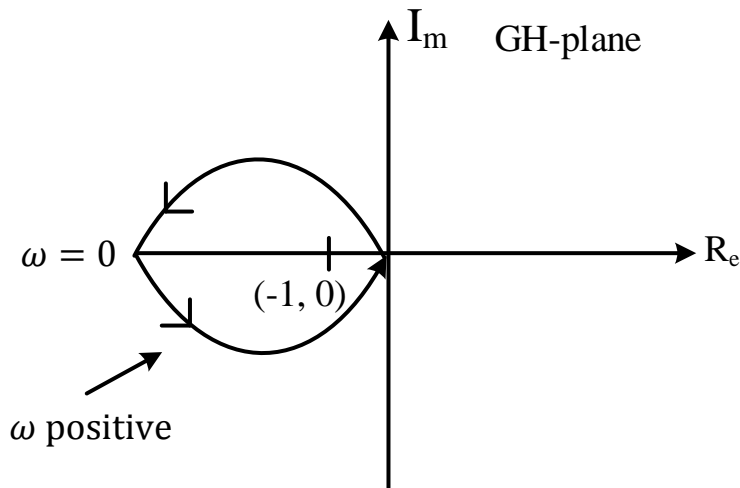
$$\text{Or } Z = P - N$$

$$= 0$$

So No roots of the characteristic equation or poles of the closed loop system lie in RH of s - plane

Option (a)

8. In the figure, the Nyquist plot of the open-loop transfer function $G(s)H(s)$ of a system is shown. If $G(s)H(s)$ has one right-hand pole, the closed-loop system is



- (a) always stable
- (b) unstable with one closed-loop right hand pole
- (c) unstable with two closed-loop right hand poles
- (d) unstable with three closed-loop right hand poles

[GATE 2003: 1 Mark]

Soln. $N = P - Z$

The encirclement of critical point $(-1, j 0)$ is in the anticlockwise direction hence $N = 1$

$P = 1$ (given)

$$Z = P - N$$

$$= 0$$

Hence no poles of closed loop system lie in the RH of $s -$ plane therefore system is always stable.

Option (a)

9. A system has poles at 0.01 Hz, 1 Hz and 80 Hz; zero at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is
- (a) -90°
 - (b) 0°
 - (c) 90°
 - (d) -180°

[GATE 2004: 2 Marks]

Soln. Phase shift are

Due to Pole at 0.01 Hz - -90°

Due to Pole at 1 Hz - -90°

Due to Pole at 80 Hz - 0

Not to be considered as the system response at 20 Hz is to be considered

Zero at 5 Hz - 90°

Zero at 100 Hz - not be considered

Zero at 200 Hz - not be considered

Thus approximate total phase shift = $-90 - 90 + 90 = -90^\circ$

Option (a)

10. The Nyquist plot of $G(j\omega)H(j\omega)$ for a closed loop control system, passed through $(-1, j 0)$ point in GH plane. The gain margin of the system in dB is equal to

(a) infinite

(c) less than zero

(b) greater than zero

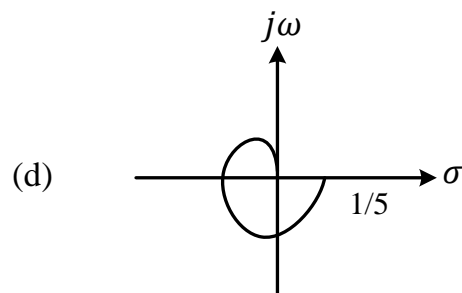
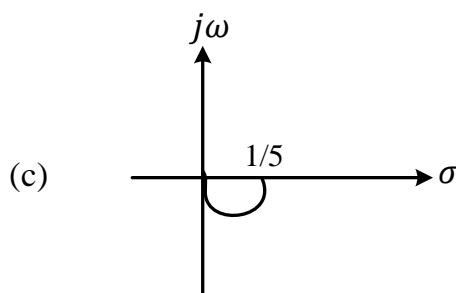
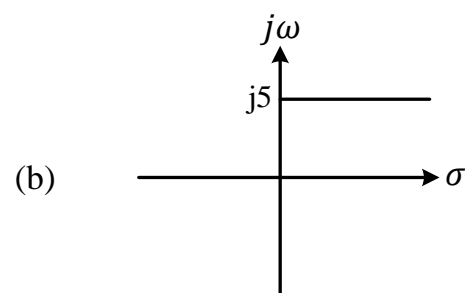
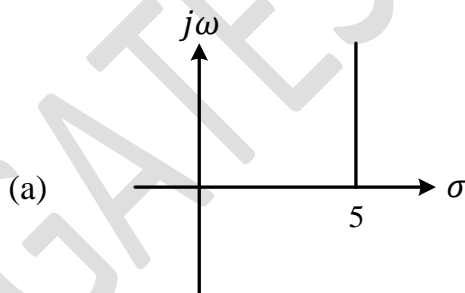
(d) zero

[GATE 2006: 2 Marks]

Soln. The gain margin of system is negative i.e. less than zero

Option (c)

11. For the transfer function $G(j\omega) = 5 + j\omega$, the corresponding Nyquist plot for positive frequency has the form



Soln. The transfer function $G(j\omega) = 5 + j\omega$

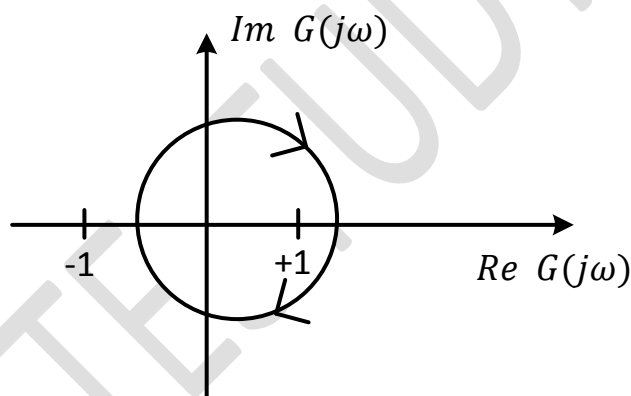
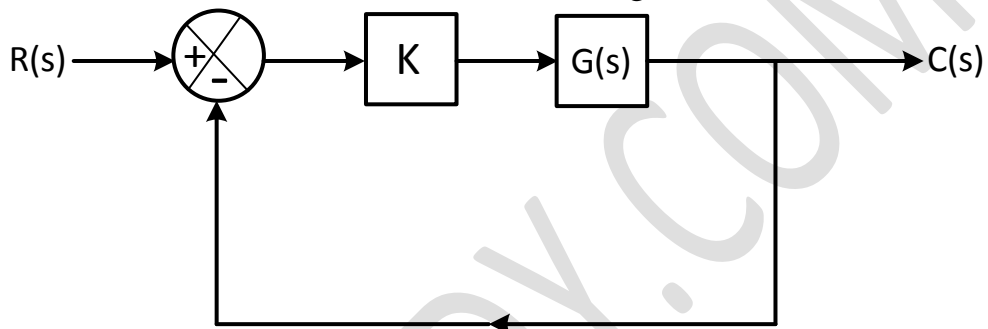
$$|G(j\omega)| = \sqrt{25 + \omega^2}$$

$$\text{At } \omega = 0 \quad |G(0)| = 5$$

$$\text{At } \omega = \infty \quad |G(\infty)| = \infty$$

Option (a)

12. Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following conclusions is correct?



- (a) $G(s)$ is an all-pass filter
- (b) $G(s)$ is strictly proper transfer function
- (c) $G(s)$ is a stable and minimum-phase transfer function
- (d) The closed-loop system is unstable for sufficiently large and positive K .

Soln. Nyquist plot is not enclosed critical point $(-1, j 0)$, hence the system is stable. If the value of gain K is increased, then intersection point moves towards $-\infty$ on the negative real axis which makes system unstable.