Frequency Response Analysis (Part – II)

1. In the Bode – plot of a unity feedback control system, the value of phase of $G(j\omega)$ at the gain cross over frequency is -125^0. The phase margin of the system is

   (a) -125^0 (c) 55^0

   (b) -55^0 (d) 125^0

   [GATE 1998: 1 Mark]

Soln. The Phase angle of $G(j\omega)$ at $\omega = \omega_{gc} = -125^0$

ω_{gc} is gain cross over frequency

$\phi_{gc} = \angle G(j\omega_{gc}) = -125^0$

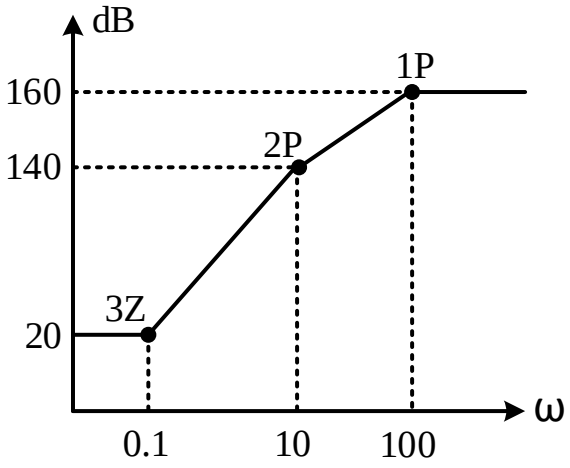
The phase margin $\gamma = 180 + \phi_{gc}$

$= 180 - 125$

$= 55^0$

Option (c)

2. The approximate Bode magnitude plot of a minimum – phase system is shown in the figure. The transfer function of the system is



(a) $10^8 \frac{(s+0.1)^3}{(s+10)^2(s+100)}$

(b) $10^7 \frac{(s+0.1)^3}{(s+10)(s+100)}$

(c) $10^8 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(d) $10^9 \frac{(s+0.1)^3}{(s+10)(s+100)^2}$

[GATE 2003: 2 Marks]

Soln. Gain changes by (140 – 20) dB when ω changes from 0.1 to 1 and 1 to 10 i.e. 2 decades slope is 60dB/decade

ω = 0.1 change in slope = +60dB/decade 3 real zero

ω = 10 change in slope is from +60dB/decade to 20dB/decade i.e. – 40dB/decade 2 real poles

ω = 100 change is slope is from 20dB/decade to 0 dB i.e. – 20dB/decade 1 finite pole

$$T.F = \frac{K\left(1+\frac{s}{0.1}\right)^3}{\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{100}\right)}$$

Magnitude is 20dB at $\omega = 0.1$

$$20\log K = 20|_{\omega=0.1}$$

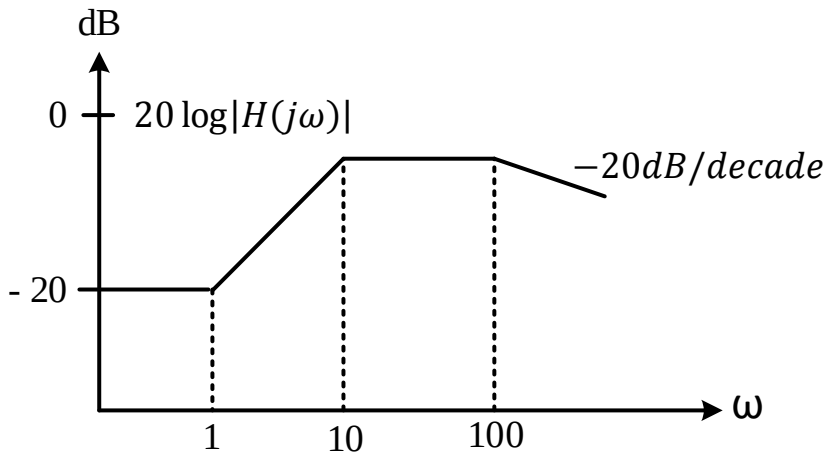
K = 10

$$T.F = \frac{10\left(1+\frac{s}{0.1}\right)^3}{\left(1+\frac{s}{0.1}\right)^3\left(1+\frac{s}{100}\right)}$$

$$= \frac{10^8(s+0.1)^3}{(s+10)^2(s+100)}$$

Option (c)

3. Consider the Bode magnitude plot shown in the figure. The transfer function H(s) is



(a) $\frac{(s+10)}{(s+1)(s+100)}$

(b) $\frac{10(s+1)}{(s+10)(s+100)}$

(c) $\frac{10^2(s+1)}{(s+10)(s+100)}$

(d) $\frac{10^3(s+100)}{(s+1)(s+100)}$

[GATE 2004: 2 Marks]

Soln. At ω = 1 change in slope 20 dB/decade

Zero at ω = 1

ω = 10, change in slope 20dB to 0dB i.e. – 20dB/decade

pole at ω = 10

ω = 100 change in slope – 20dB/decade

pole at ω = 100

Transfer function

$$H(s) = \frac{K(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

$$20\log K = -20$$

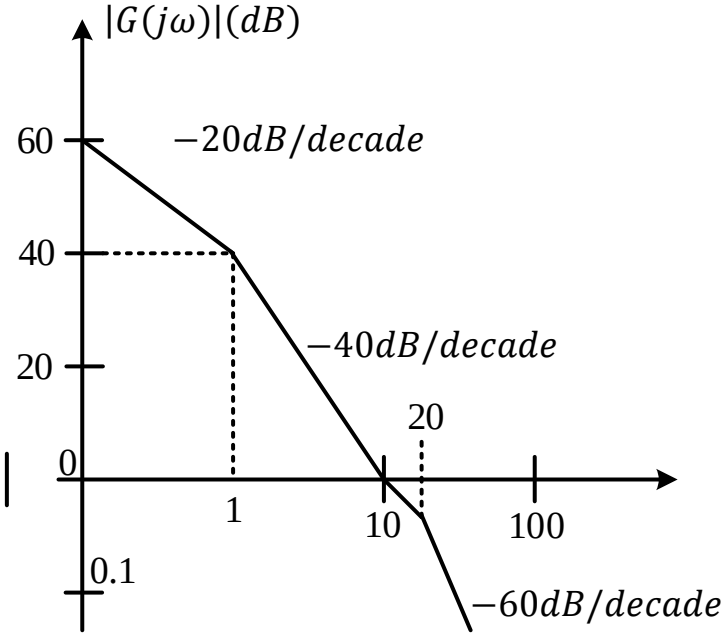
$$K = (10)^{-1} = 0.1$$

$$H(s) = \frac{0.1(s+1)\times 10^3}{(s+10)(s+100)}$$

$$= \frac{100(s+1)}{(s+10)(s+100)}$$

Option (c)

4. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function G(s) corresponding to this Bode plot is



(a) $\frac{1}{(s+1)(s+20)}$

(b) $\frac{1}{s(s+1)(s+20)}$

(c) $\frac{100}{s(s+1)(s+20)}$

(d) $\frac{100}{s(s+1)(1+0.05s)}$

[GATE 2007: 2 Marks]

Soln. Pole at s = 0, s = 1, s = 20

$$T.F = \frac{K}{s\left(1+\frac{s}{1}\right)\left(1+\frac{s}{20}\right)}$$

$60 = 20\log\frac{K}{\omega}|_{\omega=0.1}$

$60 = 20log_{10}K - 20log_{10}\omega,\ \omega = 0.1$

$60 = 20log_{10}K + 20$

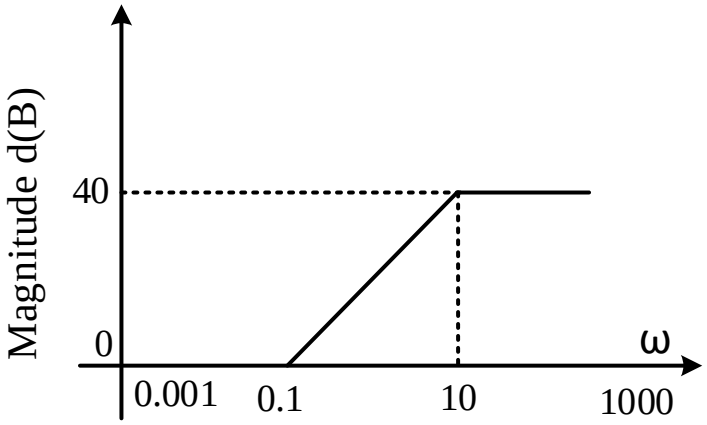
$40 = 20\ log_{10}K$

K = 100

$$T.F = \frac{100}{s(1+s)(1+0.05s)}$$

Option (d)

5. For the asymptotic Bode magnitude plot shown below, the system transfer function can be



(a) $\frac{10s+1}{0.1s+1}$

(b) $\frac{100s+1}{0.1s+1}$

(c) $\frac{100s}{10s+1}$

(d) $\frac{10s+1}{0.1s+1}$

[GATE 2010: 1 Mark]

Soln. Slope of asymptote changes from 0 to 40 dB in 2 decades (0.1 to 1, 1 to 10) or slope is 20dB/decade

Zero at ω = 0.1

ω = 10, slope changes from 20dB to 0dB i.e. (-20dB/decade)

pole at ω = 10

$$T.F = \frac{K\left(1+\frac{s}{0.1}\right)}{\left(1+\frac{s}{10}\right)}$$

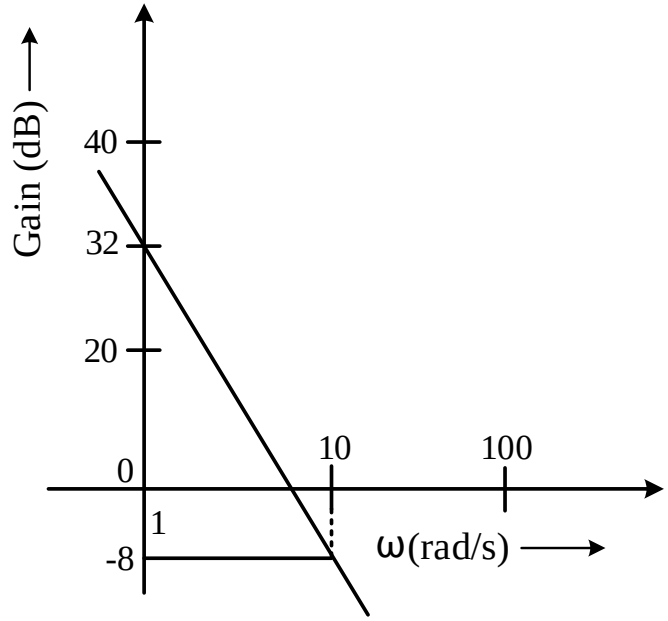
$0dB|_{\omega=0.001} = 20log_{10}K$

K = 1

$$T.F = \frac{10s + 1}{0.1s + 1}$$

Option (d)

6. The Bode plot of a transfer function G(s) is shown in the figure below:



(a) $\frac{39.8}{s}$ (c) $\frac{32}{S}$

(b) $\frac{39.8}{S^2}$ (d) $\frac{32}{S^2}$

[GATE 2013: 1 Mark]

Soln. Slope = -40dB/decade

2 poles at $\omega = 0$

$20 \log K - 40log_{10}^{10} = -8$

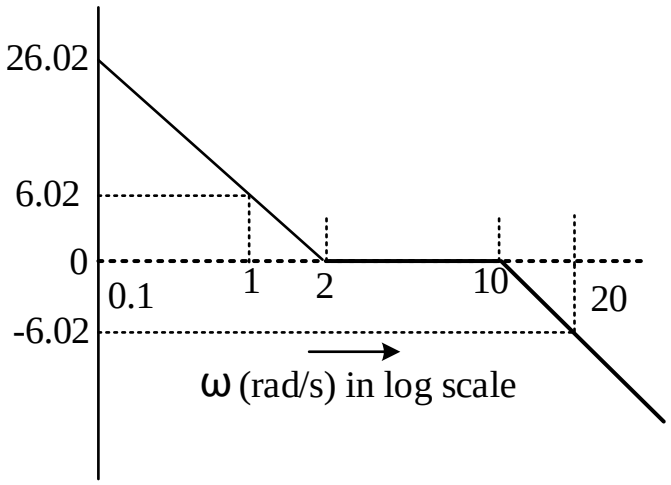
$20log_{10}^{K} = 32$

$K = 10^{32/20} = 39.8$

$T.F = \frac{39.8}{s^2}$

Option (d)

7. The Bode asymptotic magnitude plot of a minimum phase system is shown in this figure.



If the system is connected in a unity negative feedback configuration, the steady state error of the closed loop system, to a unit ramp input, is ____.

[GATE 2014: 2 Marks]

Soln. Pole at s = 0, s = 10

Zero at s = 2

The open loop transfer function of the system

$$G(s)H(s) = \frac{K\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{10}+1\right)}$$

$$26.02|_{\omega=0.1} = 20log_{10}K - 20log_{10}^{\omega}$$

$$26.02 = 20log_{10}K - 20log_{10}10^{-1}$$

$$= 20log_{10}K + 20$$

$$6.02 = 20log_{10}K$$

$$K = 1.99 \cong 2$$

$$G(s)H(s) = \frac{2 \times 10(s+2)}{2s(s+10)}$$

$$= \frac{10(s+2)}{s(s+10)}$$

Steady state error for ramp input is $e_{ss} = \frac{1}{K_a}$

$where\ K_a = \lim_{s \to 0} S\ G(s)H(s)$

$e_{ss} = \frac{1}{2} = 0.5$

8. In a Bode magnitude plot, which one of the following slopes would be exhibited at high frequency by 4th order all-pole system?
   (a) – 80 dB/decade (c) + 40 dB/decade
   (b) – 40 dB/decade (d) + 80 dB/decade

[GATE: 2014 1 Mark]

Soln. 4th order all-pole system means that the system must be having no zero or s-term in the numerator and S^4 term in denominator.

$$H(s) \propto \frac{1}{S^4}$$

One pole exhibits slope of (-20dB/decade), so four pole exhibits slope of -80dB/decade

Option (a)