Waveguides – GATE Problems

One Mark Questions

1. The interior of $a \frac{20}{3} cm \times \frac{20}{4} cm$ rectangular waveguide is completely filled with a dielectric of $\in_r = 4$. Waves of free space wave – length shorter than………..can be propagated in the TE_{11} mode.

[GATE: 1994: 1 Mark]

Soln. The inside dimension of waveguide is given.

For Rectangular waveguide

$$a = \frac{20}{3} cm \quad , \quad b = \frac{20}{4} cm$$

Where 'a' and 'b' are wide and narrow dimensions of the waveguide.

Waveguide is filled with dielectric of $\in_r = 4$

Velocity of propagation, $v = \frac{1}{\sqrt{\mu \in}}$

Cut off frequency for rectangular waveguide is given by

$$f_c = \frac{v}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Where m and n are half wave variations in wide and narrow dimensions of waveguide.

Velocity of propagation in dielectric is given by

$$v = \frac{1}{\sqrt{\mu_0 \in_0}} \frac{1}{\sqrt{\in_r}} = 3 \times 10^8 . \frac{1}{\sqrt{4}} = 1.5 \times 10^8 \frac{m}{s} = 1.5 \times 10^{10}\ cm/s$$

Since mode is TE_{11}

Thus m = 1 and n = 1

$$f_c = \frac{1.5 \times 10^{10}}{2} \sqrt{\left(\frac{3}{20}\right)^2 + \left(\frac{4}{20}\right)^2} \times 10^2\ Hz$$

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$$= 0.75 \times 10^{10} \times \frac{5}{20} = 1.875 \times 10^{9}\ Hz$$

$$= 1.875\ GHz$$

Cut off wavelength λ_C in the medium can be written as

$$\lambda_C = \frac{v}{f_c} = \frac{1.5 \times 10^{10}}{1.875 \times 10^{9}}$$

$$= 8\ cm$$

Thus the waves with free space wavelengths shorter than 8 cm wavelength can be propagated,

$$\lambda_c = 8 \text{ cm}$$

2. A rectangular air – filled waveguide has a cross section of 4 $cm \times 10\ cm$ The minimum frequency which can propagation in the waveguide is
   (a) 1.5 GHz
   (b) 2.0 GHz
   (c) 2.5 GHz
   (d) 3.0 GHz

[GATE 1997: 1 Mark]

Soln. Given,

A rectangular waveguide air filled with

a = 10 cm , b = 4 cm

Waveguide acts as a high pass filter with cut off frequency of

$$f_c = \frac{1}{2\pi\sqrt{\mu \in}} \cdot \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

For air filled waveguide

$$c = \frac{1}{\sqrt{\mu_0 \in_0}} = 3 \times 10^{8}\ m/sec$$

Here m and n are integers representing TE_{mn} or TM_{mn} modes.

Least values of m and n for TE mode are

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m=1 and n=0 representing TE_{10} mode, which is known as dominant mode having largest wavelength and lowest cut off frequency.

Thus,

$$f_c = 1.5 \times 10^8 \sqrt{\frac{1}{100} \times 10^4}\ Hz$$

$$= 1.5\ GHz$$

Option (a)

3. Indicate which one of the following modes do NOT exist in a rectangular resonant cavity

(a) TE_{110}
(b) TE_{011}
(c) TM_{110}
(d) MT_{111}

[GATE 1999: 1 Mark]

Soln. In the present problem we have to find the modes TE_{mnp} / TM_{mnp} that do not exist in rectangular cavity with dimensions a, b and d.

The integer m, n and p represent half wave variations in x, y and z directions.

For TE_{mnp} mode

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

Dominant mode present is TE_{101}

For TM_{mnp}

$$E_z = E_{0z} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{d}z\right)$$

TM mode with lowest integer present is TM_{110}

The TE_{110} mode does not exist in rectangular cavity resonator as the lowest value of last subscript should be 1 for the TE mode to exist

Option (a)

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4. The phase velocity of waves propagation in hollow metal waveguide is
   (a) Greater than velocity of light in free space
   (b) Less than velocity of light in free space
   (c) Equal to velocity of light in free space
   (d) Equal to group velocity

[GATE 2001: 1 Mark]

Soln. **The velocity of propagation in the bounded medium (i.e. say a rectangular waveguide) is given by**

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Where c – velocity of propagation in unbounded media (free space)

f_c – cut off frequency

f – Frequency of operation

Say, for TE_{10} mode

$$f > f_c$$

From the above equation. We find

$$v_p > c$$

Phase velocity is the velocity with which wave propagates. Note that the velocity with which energy propagates in waveguide is less than phase velocity.

Option (c)

5. The dominant mode in a rectangular waveguide is TE_{10} because this mode has
   (a) No attenuation
   (b) No cut off
   (c) No magnetic field component
   (d) The highest cut off wavelength

[GATE 2001: 1 Mark]

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Soln. **The dominant mode in rectangular waveguide is TE_{10} . This mode has the lowest cutoff frequency (f_c) and the highest cutoff wavelength $\lambda_C (= 2a)$ out off all the TE_{mn} and TM_{mn} modes.**

Option (d)

6. The phase velocity for the TE_{10} mode in an air filled rectangular waveguide is
   (a) Less than c
   (b) Equal to c
   (c) Greater than c
   (d) None of the above

[GATE 2002: 1 Mark]

Soln. **This problem is similar to problem of GATE 2001 (problem 4)**

Phase velocity

$$(v_p) > c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= 3 \times 10^8 \ m/s$$

Where c is velocity of plane waves in free space

Option (c)

7. The phase velocity of an electrometric wave propagating in a hollow metallic rectangular waveguide in the TE_{10} mode is
   (a) Equal to its group velocity
   (b) Less than velocity of light in free space
   (c) Equal to the velocity of light in free space
   (d) Greater than the velocity of light in free space

[GATE 2004: 1 Mark]

Soln. **This problem is also similar to problem No. 4 and 5**

Phase velocity is greater than velocity of light in free space

Option (d)

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8. Refractive index of glass is 1.5 Find the wavelength of a beam of light with a frequency of 10^{14} Hz in glass. Assume velocity of light $3 \times 10^8\ m/s$ is vacuum.

(a) 3 μm (c) 2 μm

(b) 3 μm (d) 1 μm

[GATE 2005: 1 Mark]

Soln. Given,

Refractive index of glass n = 1.5

Frequency of light beam $f = 10^{14}\ Hz$

Velocity of light in Vacuum = $c = 3 \times 10^8\ m/s$

Refractive index of medium is given by

$$n = \frac{c}{v} = \frac{Velocity\ of\ em\ wave\ in\ free\ space}{Velocity\ of\ light\ in\ medium}$$

$$or, \qquad v = \frac{c}{n} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ m/s$$

$$v = f\lambda \quad or, \quad \lambda = \frac{v}{f} = \frac{2 \times 10^8}{10^{14}} = 2 \times 16^{-6}\ m = 2\ \mu m$$

Option (c)

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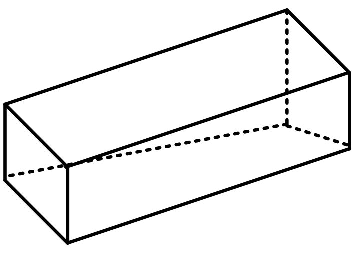
9. Which of the following statement is true regarding the fundamental mode of the metallic waveguides shown

P: Coaxial

Q: Cylindrical



R: Rectangular

(a) Only P has no cut off frequency
(b) Only Q has no cut off frequency
(c) Only R has no cut off frequency
(d) All three have cut off frequency

[GATE 2009: 1 Mark]

Soln. Transmission media such as

(i) Two wire line
(ii) Coaxial line
(iii) Parallel plane waveguide (for TM modes) having two plates as conductor

These media have no cut off frequency $(f_c = 0)$.The wave propagated is called principal wave. This is also known as Transverse Electromagnetic wave (TEM) wave.

Thus, Coaxial line shown in P has no cut off frequency

$$i.e.\ \ f_c = 0$$

Cylindrical waveguide shown in figure Q and rectangular waveguide shown in R, are the single conductor system having cut off frequency, f_c, which depends upon the dimensions of the cross section and characteristic of the medium in the wave guide.

Option (a)

10. The modes of rectangular waveguide are denoted by TE_{mn} / TM_{mn} when m and n are Eigen numbers along the larger and smaller dimensions of the waveguide respectively. Which one of the following statement is true.
(a) The TM_{10} mode of waveguide does not exist.
(b) The TE_{10} mode of waveguide does not exist.
(c) The TM_{10} and TE_{10} modes both exist and have same cut off frequency.
(d) The TM_{10} and TE_{10} modes both exist and have same cut off frequency

[GATE 2011: 1 Mark]

Soln. In a rectangular waveguide the lowest value of m or n for TM mode is unity So the lowest TM mode is TM_{11} (TM_{01} or TM_{10} modes do not exist.)

For TE mode, TE_{10} and TE_{01} modes exist.

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The lowest order TE mode is TE_{10} . This mode has the lowest cut off frequency and is called the dominant mode.

If we look to various options given we find

Option (a)

11. Consider an air filled rectangular waveguide with a cross – section of 5 cm × 3 cm. For this waveguide, the cut off frequency (in MHz) of TE_{21} mode is ________

[GATE 2014: 1 Mark]

Soln. For air filled rectangular waveguide the cut off frequency is given by

$$f_c = \frac{1}{2\pi\sqrt{\mu_0 \in_0}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

For TE_{21} mode

$m = 2 \quad and \quad n = 1, \quad a = 5cm, \quad b = 3cm$

f_c for TE_{21} mode is given by

$$f_{c21} = \frac{3\times 10^8}{2}\sqrt{\left(\frac{2}{0.05}\right)^2 + \left(\frac{1}{0.03}\right)^2}$$

$$f_{C21} = 7810\ MHz$$

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Two Mark Questions

1. The cut off frequency of waveguide depends upon
   (a) The dimensions of the waveguide.
   (b) The dielectric property of the medium in the waveguide.
   (c) The characteristic impedance of the waveguide
   (d) The transverse and axial components of the fields

[GATE 1987: 2 Marks]

Soln. Let us consider the expression for cut off frequency for rectangular waveguides.

$$f_c = \frac{1}{2\pi\sqrt{\mu \in}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Thus, it depends on

(i) Dimension a and b of the waveguide
(ii) Dielectric constant of the medium

Thus, option (a) and (b)

2. For normal mode EM wave propagation in a hollow rectangular waveguide
   (a) The phase velocity is greater than group velocity.
   (b) The phase velocity is greater than velocity of light in free space.
   (c) The phase velocity is less than the velocity of light in free space.
   (d) The phase velocity may be either greater than or less than group velocity.

[GATE 1988: 2 Marks]

Soln. In a rectangular waveguide the phase velocity is (v_p)

$v_p >$ velocity of light in free space and is given by

$$v_p = \frac{c}{\sqrt{1-(f_c/f)^2}}$$

Where f_c is cut off frequency.

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The group velocity v_g in the guide is related to v_p and c

$$v_p \, . v_g = c^2$$

$$v_g = \frac{c^2}{v_p} \quad or \quad v_g = c . \sqrt{1 - (f_c/f)^2}$$

$$or \quad v_g < c$$

Therefore $\quad v_g < c < v_p$

Thus,

Options (a) and (d)

3. Choose the correct statements for a wave propagating in an air filled rectangular waveguide
   (a) Guided wavelength is never less than free space wavelength.
   (b) Wave impedance is never less than free space impedance.
   (c) Phase velocity is never less than the free space velocity.
   (d) TEM mode is possible if the dimensions of the waveguide are properly chosen.

[GATE 1990: 2 Marks]

Soln. For the wave propagating in air filled rectangular waveguide

Guide wavelength is given by

$$\lambda_g = \frac{2\pi}{\beta} = \frac{\lambda_0}{1 - (\lambda_0/\lambda_C)^2} = \frac{\lambda_0}{\sqrt{1 - (f_c/f)^2}}$$

Where λ_0 – free space wavelength

Thus

$$\lambda_C > \lambda_0$$

Wave impedance for TE wave is given by

$$\eta_{TE} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}$$

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Where η is impedance in free space for TEM waves

Thus,

$$\eta_{TE} > \eta$$

For TM waves

$$\eta_{TM} = \eta\sqrt{1-(f_c/f)^2}$$

$$\eta_{TM} < \eta$$

Phase velocity

$$v_p = \frac{\omega}{\beta} = \frac{1}{\mu \in} \times \frac{1}{\sqrt{1-(f_c/f)^2}}$$

$$= \frac{c}{\sqrt{\mu_r \in_r}\sqrt{1-(f_c/f)^2}}$$

Where c is velocity in free space

$$v_p > c$$

Options (a) and (c)

4. A rectangular waveguide has dimensions $1cm \times 0.5\ cm$. Its cut off frequency is

(a) 5 GHz
(b) 10 GHz
(c) 15 GHz
(d) 20 GHz

[GATE 2000: 2 Marks]

Soln. Dimensions of rectangular waveguide

$$a = 1\ cm = 10^{-2}m$$

$$b = 0.5\ cm = 0.5 \times 10^{-2}m$$

The lowest possible mode is TE_{10}

Cut off frequency (f_c) for TE_{10} mode is

$$f_c = \frac{c}{2a}$$

Where c is velocity of light free space

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$$f_c = \frac{3 \times 10^{10}}{2 \times 10^{-2}} = 1.5 \times 10^{10}$$

$$= 15\ GHz$$

Option (c)

5. A rectangular metal wave guide filled with a dielectric material of relative permittivity $\varepsilon_r = 4$ has the inside dimensions $3.0\ cm \times 1.2\ cm$. The cut off frequency for the dominant mode is

(a) 2.5 GHz (c) 10.0 GHz
(b) 5.0 GHz (d) 12.5 GHz

[GATE 2003: 2 Marks]

Soln. Given,

Waveguide dimensions (inner)

$$a = 3\ cm = 3 \times 10^{-2} m$$

$$b = 1.2\ cm = 1.2 \times 10^{-2} m$$

Dominant mode is the mode having lowest cut off frequency and is denoted by TE_{10} .Cut off frequency for Dominant mode is given by

$$f_c = \frac{v}{2a}$$

Where v is velocity in the medium

$$f_c = \frac{v}{2a}$$

$$= \frac{1}{\sqrt{\mu_r \in_r}} \cdot \frac{1}{2a}$$

$$= \frac{1}{\sqrt{\mu_0 \in_0}\sqrt{\in_r}} \cdot \frac{1}{2a}$$

$$= \frac{3 \times 10^8}{\sqrt{4} \times 2 \times 3 \times 10^{-2}}$$

$$= 0.25 \times 10^{10}$$

$f_c = 2.5\ GHz$ **Option (a)**

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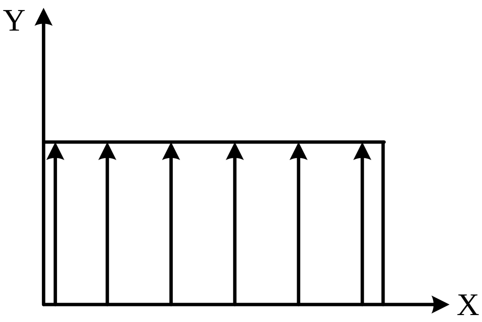
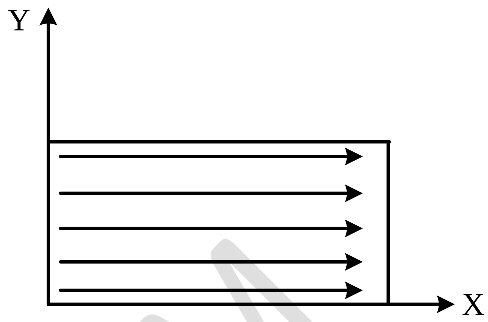
6. Which one of the following does represent the electric field lines for the TE_{02} mode in the cross – section of a hollow rectangular metallic waveguide?

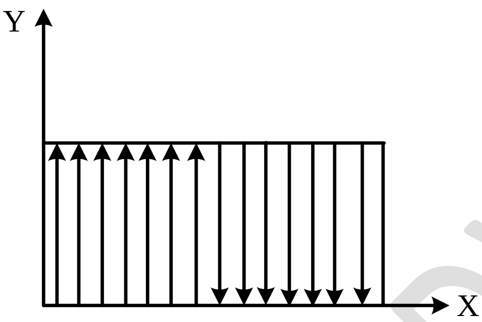
(a)



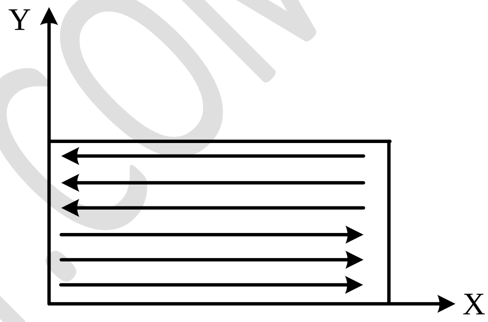
(b)



(c)



(d)



[GATE 2005: 2 Marks]

Soln. This problem is to find E-field configuration in rectangular waveguide for TE_{02} mode.

The first subscript m = 0 indicates no variation of E in x – direction.

The second subscript n = 2 indicates two – half wave variations in y – direction.

This variation agrees with option (d) shown in figure

Option (d)

7. A rectangular waveguide having TE_{10} mode as dominant mode is having a cut off frequency of 18 GHz for the TE_{30} mode. The inner broad – wall dimension of the rectangular waveguide is

(a) 5/3 cm (c) 5/2 cm
(b) 5 cm (d) 10 cm

[GATE 2006: 2 Marks]

Soln. Cut off frequency for TE_{mn} mode is

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$$f_c = \frac{1}{2\sqrt{\mu \in}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For waveguide with air medium

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Here, for TE$_{30}$ mode

$$m = 3, n = 0$$

$$f_c = \frac{c}{2}\cdot\frac{3}{a}$$

$$So, \quad a = \frac{c}{2}\cdot\frac{3}{f_c}$$

$$= \frac{3\times 10^8}{2}\cdot\frac{3}{18\times 10^9}$$

$$= \frac{5}{2}\ cm$$

Option (c)

8. An air – filled rectangular waveguide has inner dimensions of $3\ cm \times 2\ cm$. The wave impedance of the TE$_{20}$ mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance $\eta_0 = 377\ \Omega$).

(a) 308 Ω
(b) 355 Ω
(c) 400 Ω
(d) 461 Ω

[GATE 2007: 2 Marks]

Soln. Given,

Inner dimension of waveguide is 3 $cm \times 2\ cm$

Free space impedance $\eta_0 = 377\ \Omega$

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Wave impedance η for TE$_{20}$ mode at $f = 30\ GHz$ is given by

$$\eta = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}}$$

$$f_c \ \ for\ TE_{20} = \frac{c}{2}\sqrt{\left(\frac{2}{3}\right)^2 + 0}$$

$$= \frac{c}{2}\times\frac{2}{3} = \frac{c}{3} = \frac{3\times 10^8}{3}$$

$$f_c = 10GHz$$

So,

$$\eta = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}} = \frac{377}{\sqrt{1-(10/30)^2}}$$

$$= \frac{\eta}{0.943} \cong 400\Omega$$

Option (c)

9. The $\vec{E}$ field in a rectangular waveguide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega\ \mu}{h^2}\left(\frac{\pi}{a}\right)\ H_0 \sin\left(\frac{2\pi x}{a}\right)\sin(\omega t - \beta z)\,\hat{y},$$

Where H_0 is a constant, a and b are the dimensions along the x – axis and the y – axis respectively. The mode of propagation in the waveguide is

(a) TE_{20}
(b) TM_{11}
(c) TM_{20}
(d) TM_{10}

[GATE 2007: 2 Marks]

Soln. Given

Wide dimensions of waveguide is a

Narrow dimensions of wave is b

Field is given by

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$$\vec{E} = \frac{\omega\,\mu}{h^2}\left(\frac{\pi}{a}\right)\; H_0 \sin\left(\frac{2\pi x}{a}\right) \sin(\omega t - \beta z)\,\hat{y},$$

Wave is traveling in +z direction (factor $-\beta z$)
The component of electric field is in y direction i.e. E_y component (as function of x)
No component of field in the direction of propagation ($\vec{a}_z$)
So, the wave is transverse electric (TE)
If we compare 'sin' term in $\vec{E}$ with general expression $sin\left(\frac{m\pi}{a}\right)x$
We find m = 2
There is no function of 'in E i.e. n=0
Thus the mode of propagation in waveguide is TE_{20}
Option (a)

10. A rectangular waveguide of internal dimensions (a = 4 cm and b = 3 cm) is to be operated in TE_{11} mode. The minimum operating frequency is

(a) 6.25 GHz
(b) 6.0 GHz
(c) 5.0 GHz
(d) 3.75 GHz

[GATE 2008: 2 Marks]

Soln. Given,

A rectangular waveguide with internal dimensions

a = 4 cm

b = 3 cm

Mode of operation is TE_{11} .We have to find the minimum operating frequency.

For any mode of operation the minimum frequency is the cut off frequency of that mode. So we have to find the cut off frequency of TE_{11} mode.

$$f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \qquad For\ TE_{11}\ \ Mode$$

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$$= \frac{3 \times 10^8}{2} \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2}$$

$$= 1.5 \times 10^{10} \times \frac{5}{12}\ Hz$$

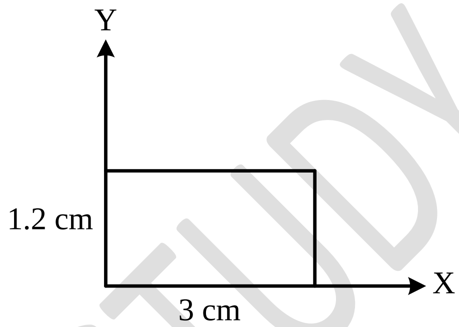
$$= 6.25\ GHz$$

Option (a)

11. The magnetic field along the propagation direction inside a rectangular waveguide with the cross section shown in the figure is

$$H_z = 3\cos(2.094 \times 10^2 x)\cos(2.618 \times 10^2 y)\cos(6.283 \times 10^{10}\ t - \beta\ z)$$

The phase velocity v_P of the wave inside the waveguide satisfies



(a) $v_P > c$
(b) $v_P = c$
(c) $0 < v_p < c$
(d) $v_P = c$

[GATE 2012: 2 Marks]

Soln. For the rectangular waveguide as per the given figure

a = 3 cm , b = 1.2 cm

$$H_z = 3\cos(2.094 \times 10^2 x)\cos(2.618 \times 10^2 y)\cos(6.283 \times 10^{10}\ t - \beta\ z)$$

From above equation

$$\omega = 6.283 \times 10^{10}\ rad/sec$$

$$f = \frac{\omega}{2\pi} = \frac{6.283 \times 10^{10}}{2\pi} = 10\ GHz$$

$$\frac{m\pi}{a} = 2.094 \times 10^2$$

$$or, \quad \frac{m}{a} = \frac{2.094 \times 10^2}{\pi} = 66.65/m$$

$$\frac{n\pi}{b} = 2.618 \times 10^2$$

$$\frac{n}{b} = \frac{2.618 \times 10^2}{\pi} = 83.33/m$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{2}\sqrt{(66.65)^2 + (83.33)^2}$$

= 16 GHz

The wave with frequency of 10 GHz will not propagate through the waveguide

Thus phase velocity of wave inside the waveguide will be 0

$$v_p = 0$$

Option (d)

12. For a rectangular waveguide of internal dimensions $a \times b (a > b)$, the cut – off frequency for the TE_{11} mode is the arithmetic mean of the cut – off frequencies for TE_{10} mode and TE_{20} mode. If $a = \sqrt{5}$ cm. the value of b (in cm) is --------.

[GATE 2014: 2 Marks]

Soln. Cut off frequency is given by

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$$f_c = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

For TE_{10} mode, m = 1 and n = 0

$$f_{c_{10}} = \frac{c}{2\pi}\cdot\left[\frac{\pi^2}{a^2}\right]^{1/2} = \frac{c}{2a}$$

$$f_{c_{20}} = \frac{c}{2\pi}\cdot\sqrt{\left(\frac{2\pi}{a}\right)^2} = \frac{c}{a}$$

Where

$$c = \frac{1}{\sqrt{\mu_0 \in_0}}$$

Given, $a = \sqrt{5}\ cm$

Arithmetic mean

$$= \frac{1}{2}\left(\frac{c}{2a} + \frac{c}{a}\right) = \frac{3c}{4a}$$

$$f_{c_{11}} = \frac{c}{2\pi}\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]^{1/2} = \frac{3c}{4\sqrt{5}}$$

$$or,\quad \frac{c}{2}\left[\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{1}{b}\right)^2\right]^{1/2} = \frac{3c}{4\sqrt{5}}$$

$$or,\quad \left[\frac{1}{5} + \frac{1}{b^2}\right]^{1/2} = \frac{3}{2\sqrt{5}}$$

$$or,\quad \frac{1}{5} + \frac{1}{b^2} = \frac{9}{4\times 5}$$

$$or,\quad \frac{1}{b^2} = \frac{9}{20} - \frac{1}{5} = \frac{9-4}{20} = \frac{5}{20} = \frac{1}{4}$$

Or, b = 2 cm

13. The longitudinal component of the magnetic field inside an air – filled rectangular waveguide made of a perfect electric conductor is given by the following expression

$$H_z(x, y, z, t) = 0.1\cos(25\pi x)\cos(30.3\,\pi y)\cos(12\pi \times 10^9 t - \beta z)\ (A/m)$$

The cross – sectional dimensions of the waveguide are given as a = 0.08 m and b = 0.033 m. The mode of propagation inside the waveguide is

(a) TM_{12}
(b) TM_{21}
(c) TE_{21}
(d) TE_{12}

[GATE 2015: 2 Marks]

Soln. In the problem longitudinal component of magnetic field is given, so the wave is transverse electric i.e. modes will be of TE type

$$E_z = 0 \quad and \quad H_z \neq 0$$

So mode will be TE_{mn}

Given,

a = 0.08m , b = 0.033m

From the given field equation

$$\frac{m\pi}{a}x = 25\,\pi x$$

$$or, \quad m = 25a$$

$$= 25 \times 0.08m$$

$$= 2$$

$$\frac{n\pi}{b}y = 30.3y$$

$$or, \quad n = (30.3) \times b = 30.3 \times 0.033$$

$$or \quad n = 1$$

So, the mode is

TE_{21}

Option (c)

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14. An air – filled rectangular waveguide of internal dimension a $cm \times b\ cm\ (a > b)$ has a cut off frequency of 6 GHz for the dominant TE_{10} mode. For the same waveguide, if the cutoff frequency of the TM_{11} mode is 15 GHz, the frequency of the TE_{01} mode GHz is ____________

[GATE 2015: 2 Marks]

Soln. Dimensions of waveguide

Wide dimension a cm

Narrow dimension b cm

Cut off frequency $(f_c) = 6\ GHz$ for dominant mode

For dominant mode cut off wavelength (λ_C) is by 2a

$$i.e.\ \lambda_{C_{10}} = 2a$$

$$or,\ \ \lambda_{C_{10}} = \frac{c}{fc_{10}} = \frac{3 \times 10^{10}}{6 \times 10^9} = 5cm$$

$$\lambda_{C_{10}} = 2a = 5cm$$

So a = 2.5 cm

fc for TM_{11} mode is 15 GHz

$$i.e.\ \ \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 15 \times 10^9$$

$$or,\ \ b = \frac{2.5}{\sqrt{5.25}}$$

Now fc for TE_{01} mode is

$$f_c = \frac{c}{2b} = \frac{3 \times 10^{10}}{2.5}\ .\sqrt{5.25}$$

= 13.75 GHz

Answer 13.75 GHz

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15. Consider an air – filled rectangular waveguide with dimensions a = 2.286 cm and b = 1.016 cm. At 10 GHz operating frequency, the value of the propagation constant (per meter) of the corresponding propagation mode is ___________

[GATE 2016: Marks]

Soln. Given,

Air filled rectangular waveguide

a = 2.286 cm

b = 1.016

f = 10 GHz

Assume dominant mode of propagation in the waveguide i.e. TE_{10} mode cut off frequency for TE_{10} mode is given by (for m = 1 and n = 0)

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2} = \frac{c}{2}\cdot\frac{1}{a}$$

$$f_c = \frac{3 \times 10^{10}}{2 \times 2.286} = 6.56\ GHz$$

Since cut off frequency is 6.56 GHz, the frequency of 10 GHz will propagate in the waveguide

Propagation constant

$$\gamma = a + j\beta$$

If $\alpha = 0$

β **is phase constant**

$$\beta = \frac{2\pi}{\lambda_g} = \frac{\frac{2\pi}{\lambda_0}}{\sqrt{1 - \left({\lambda_0}/{\lambda_C}\right)^2}} = \frac{2\pi}{\lambda_0}\sqrt{1 - \left({\lambda_0}/{\lambda_C}\right)^2}$$

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$$= \frac{2\pi}{c/f}\sqrt{1 - \left({fc}/{f}\right)}$$

$$= 2\pi \cdot \frac{f}{c}\sqrt{1 - \left({fc}/{f}\right)^2}$$

$$= 2\pi \times \frac{10 \times 10^9}{3 \times 10^8}\sqrt{1 - \left(\frac{6.56}{10}\right)^2}$$

$$= \frac{2\pi}{3} \times 100 \times 0.5696 = \frac{2\pi}{3} \times 56.96$$

$$\beta = 158\, m^{-1}$$

16. Consider an air – filled rectangular waveguide with dimensions a = 2.286 cm and b = 1.016 cm. The increasing order of the cut – off frequency for different modes is

(a) $TE_{01} < TE_{10} < TE_{11} < TE_{20}$
(b) $TE_{20} < TE_{11} < TE_{10} < TE_{01}$
(c) $TE_{10} < TE_{20} < TE_{01} < TE_{11}$
(d) $TE_{10} < TE_{11} < TE_{20} < TE_{01}$

[GATE 2016: 2 Marks]

Soln. Waveguide dimensions are

a = 2.286 cm

b = 1.016 cm

waveguide is air filled

cut off frequency $(f_c) = \frac{C}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{m}{b}\right)^2}$

$$fc\ for\ TE_{11}\ mode = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{3 \times 10^{10}}{2}\sqrt{\frac{1}{(2.216)^2} + \frac{1}{(1.016)^2}}$$

= 16.15 GHz

$$f_c\ for\ TE_{01}\ mode = \frac{c}{2b} = \frac{3 \times 10^{10}}{2 \times 1.016} = 14.76\ GHz$$

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$$f_c \; for \; TE_{20} \; mode = \frac{c}{a} = \frac{3 \times 10^{10}}{2.286} = 12.12 \; GHz$$

$$f_c \; for \; TE_{10} \; mode = \frac{c}{2a} = \frac{3 \times 10^{10}}{2 \times 2.286} = 6.56 \; GHz$$

Thus we find

Cut off frequency is given by

$TE_{10} < TE_{20} < TE_{01} < TE_{11}$

Thus, Option (c)

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