Boolean algebra and Logic Simplification

Key point

The first two problems at S. Nos. 1 and 2 are on the Number of Boolean expressions for a given number of variables.

The number of Boolean expressions for n variables is $2^{2^n}$

Note that for n variable Boolean function one can have $2^n$ Boolean inputs.

1. The number of Boolean function that can be generated by n variables is equal to:
   (a) $2^{2^n}$
   (b) $2^n$
   (c) $2^{n-1}$
   (d) $2^n$  
   
   [GATE 1990 : 1 Mark]

   Ans. (b)

   The number of district Boolean expressions for n variables is $2^{2^n}$

   Where, n is the number of variables.

2. The number of distinct Boolean expressions of 4 variables is
   (a) 16
   (b) 256
   (c) 1024
   (d) 65536

   [GATE 2003 : 1 Mark]

   Ans. (d)

   We know that the number of Boolean expressions for n variables is $2^{2^n}$

   So, for $n = 4$

   $2^{2^4} = 2^{16} = 65536$

3. Two 2’s complement number having sign bits x and y are added and the sign bit of the result is z. Then, the occurrence of overflow is indicated by the Boolean function.
   (a) $x \cdot y \cdot z$
   (b) $\bar{x} \cdot \bar{y} \cdot \bar{z}$
   (c) $\bar{x} \cdot \bar{y} \cdot z + x \cdot y \cdot \bar{z}$
   (d) $x y + yz + zx$

   [GATE 1998 : 1 Mark]
Ans. (c)

This problem is based on 2’s complement addition/subtraction (covered in number systems). Here we have to write the Boolean expression for the case where overflow occurs.

In the 2’s complement arithmetic we had seen that if MSB (Sign bit) of operands (Minuend & Subtrahend) is same, and the MSB of the result is different or vice versa an overflow occurs.

i.e.  
\[ x = 0, \quad y = 0 \quad \text{and} \quad z = 1 \]

or, if  
\[ x = 1, \quad y = 1 \quad \text{and} \quad z = 0 \]

Overflow occurs

So, we can write

\[ \overline{x} \overline{y} \overline{z} + x y \overline{z} \]

is the Boolean function for the overflow to occur.

Key points:

The problems in Q Nos. 3, 4, 5, 6, 7 and 8 are on the simplification of Boolean expressions based on algebraic methods like using laws and theorems of Boolean algebra. But this method becomes complex with increase in number of variables and number of terms.

4. The logical expression \( y = A + \overline{A}B \) is equivalent to
   (a) \( y = AB \)  
   (b) \( y = \overline{A}B \)  
   (c) \( y = \overline{A} + B \)  
   (d) \( y = A + B \)

   [GATE 1999 : 1 Mark]

Ans. (d)

\[
\begin{align*}
y & = A + \overline{A}B \\
& = (A + \overline{A})(A + B) \\
& = 1 \cdot (A + B) = (A + B)
\end{align*}
\]

5. The minimized form of the logical expression \((\overline{A} \overline{B} \overline{C} + \overline{A}B \overline{C} + \overline{A}B \overline{C} + AB \overline{C})\) is
   (a) \( \overline{A} \overline{C} + B \overline{C} + \overline{A}B \)  
   (b) \( \overline{A} \overline{C} + \overline{B} \overline{C} + \overline{A}B \)  
   (c) \( \overline{A} \overline{C} + \overline{B} \overline{C} + \overline{A}B \)  
   (d) \( A \overline{C} + \overline{B} \overline{C} + A \overline{B} \)

   [GATE 1999 : 2 Marks]

Ans. (a)

\[
\overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + \overline{A}B \overline{C} + \overline{A}B \overline{C}
\]

Combining first two terms
\[ \overline{A}C(B + B) + \overline{A}BC + AB\overline{C} \]
\[ = \overline{A}C + \overline{A}BC + AB\overline{C} \]
\[ = \overline{A}(\overline{C} + BC) + AB\overline{C} \]
\[ = \overline{A}(\overline{C} + C)(\overline{C} + B) + AB\overline{C} \]
\[ = \overline{A}(\overline{C} + B) + AB\overline{C} \]
\[ = \overline{A}\overline{C} + AB + AB\overline{C} \]
\[ = \overline{A}\overline{C} + B(\overline{A} + \overline{A}\overline{C}) \]
\[ = \overline{A}\overline{C} + B(\overline{A} + \overline{C}) \]
\[ = \overline{A}\overline{C} + BC + AB \]

6. The Boolean expression \( AC + B\overline{C} \) is equivalent to
(a) \( \overline{A}C + B\overline{C} + AC \)
(b) \( B\overline{C} + AC + B\overline{C} + \overline{A}\overline{C} \overline{B} \)
(c) \( AC + B\overline{C} + \overline{B}C + ABC \)
(d) \( ABC + \overline{A}B\overline{C} + AB\overline{C} + A\overline{B}\overline{C} \)

[\text{GATE 2004 : 2 Marks}]

Ans. (d)

\[ AC + B\overline{C} \]
\[ = AC(B + \overline{B}) + \overline{B}C(A + \overline{A}) \]
\[ = ABC + \overline{AB}C + \overline{AB}\overline{C} + \overline{AB}C \]
\[ = ABC + \overline{AB}C + \overline{AB}C + \overline{AB}C \]

7. The Boolean expression for the truth table shown is

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) \( B(A + C)(\overline{A} + \overline{C}) \)
(b) \( B(A + \overline{C})(\overline{A} + C) \)
(c) \( \overline{B}(A + \overline{C})(\overline{A} + C) \)
(d) \( \overline{B}(A + B)(\overline{A} + \overline{C}) \)
We observe that the expression needed in the result is of POS form but in truth table the output is 0 for many terms but 1 only for 2 terms so we write the SOP equation. Then convert it to POS

\[ F = \overline{A}BC + AB\overline{C} \]
\[ = B(\overline{A}C + A\overline{C}) = B. (A \oplus C) \]
\[ = B(A + C)(\overline{A} + \overline{C}) \]

Since
\[ A \oplus C = A \overline{C} = AC + \overline{AC} \]
So, \[ A \overline{C} = AC + \overline{AC} \]
\[ = (\overline{A} + \overline{C})(A + C) \]
\[ (A \oplus C) \]

8. If \( X = 1 \) in the logic equation

\[ \{X + Z(\overline{Y} + (\overline{Z} + X \overline{Y}))\} \{X + Z(X + Y)\} = 1 \]
then

(a) \( Y = Z \)
(b) \( Y = \overline{Z} \)
(c) \( Z = 1 \)
(d) \( Z = 0 \)

An ans. (d)

Given logic equation is

\[ \{X + Z(\overline{Y} + (\overline{Z} + X \overline{Y}))\} \{X + Z(X + Y)\} = 1 \]

\( X = 1 \), so the first term is \( \{1 + \cdots \} = 1 \)

2nd term is \( \{0 + \overline{Z}(1 + Y)\} = \overline{Z} \)

1. \( \overline{Z} = 1 \) or \( Z = 0 \)

Key Points:

- Prime implicants: A product term in SOP which cannot be further simplified by combination with other terms.
- SOP (Sum of product) and POS (produser Sum) expressions.
- K-Map has \( 2^k \) cells where \( k \) is no. of variables.
- Don’t care combinations: Represented as ‘d’. d’s can be combined with 1’s to simplify, where needed.
9. The K-map for a Boolean function is shown in figure. The number of essential prime implicants for this function is

(a) 4  
(b) 5  
(c) 6  
(d) 8  

[GATE 1998 : 1 Mark]

Ans. (a)

On the K-Map retaining only minimum terms (corresponding to 1)

The number of prime implicants are

\[ \overline{ACD} + ABC + ABC + BD \]

Four prime implicants

10. The number of products terms in the minimized sum-of-product expression obtained through the following K-map is (where, “d” denotes don’t care states)
The Boolean expression $Y = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}D + A\overline{B}\overline{C}D$ can be minimized to

(a) $Y = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C} + A\overline{C}D$  
(b) $Y = \overline{A}\overline{B}\overline{C}D + B\overline{C}D + A\overline{B}\overline{C}D$

An. (d)
The Boolean expression
\[ Y = \overline{A}BCD + \overline{A}BCD + A\overline{B}CD + A\overline{B}C \]
Min terms can be plotted on K-Map
The minimized expression is
\[ Y = \overline{A}BCD + ABCD + BCD \]
Alternative method
\[ Y = \overline{A}BCD + \overline{A}BCD + A\overline{B}CD + A\overline{B}C \]
Combining first and third terms
\[ = (\overline{A} + A)\overline{B}CD + A\overline{B}CD + A\overline{B}C \]
\[ = \overline{A}BCD + ABCD + BCD \]

12. In the sum of products function \( f(X, Y, Z) = \sum(2,3,4,5) \), the prime implicants are
(a) \( \overline{X}Y, XY \)  
(b) \( \overline{X}Y, XYZ, \overline{X}Z \)  
(c) \( \overline{X}YZ, \overline{X}YZ, XY \)  
(d) \( \overline{X}YZ, \overline{X}YZ, XY \overline{Z}, X\overline{Y} \)

Ans. (a)

\[ f = (X, Y, Z) = \sum(2, 3, 4, 5) \]
\[ = X\overline{Y} + \overline{X}Y \]
So prime implicants are
\( X\overline{Y} \) and \( \overline{X}Y \)