Network Theorems (Part I)-Numerical Problems

Key points: - The problems considered in this set are involving both dependent and independent sources. Following points may be noted

- Dependent sources are voltage or current sources whose output is function of another parameter in the circuit.
- Dependent sources only produce a voltage or current when an independent voltage or current source is in the circuit.
- Dependent sources are treated like independent sources when using nodal or mesh analysis but not with superposition.

1. In the following circuit find the value of $v_{TH}$ and $R_{TH}$

![Circuit Diagram]

Ans. $V_{th} = \frac{8}{(2+6)} \times 6 = 6V$

$R_{th} = (2||6) + 2 = 3.5\Omega$ (Replacing voltage source by it’s internal impedance)

2. In the following circuit find the value of $i_N$ and $R_N$

![Circuit Diagram]
Ans. \( R_N = 2 \parallel 4 + 2 \)
\[
= \frac{10}{3} \Omega
\]

\( i_N \) = short circuit current \( (i_{sc}) \), By source transformation the equivalent circuit is

\[
i_{sc} = \frac{7.5 \times (2 \parallel 4)}{2 + 2 \parallel 4} = \frac{7.5 \times 4}{\frac{10}{3}} = \frac{30}{10} = 3 \text{ amps}
\]

3. In the following circuit find the value of \( v_{TH} \) and \( R_{TH} \)

Ans. Current through 1\( \Omega \) resistor (using current division) = \( \frac{2 \times 3}{3 + 3} = 1 \text{ amp} \)

\( V_{th} = 1 \text{ volt} \)

\( R_{th} = 1 \parallel 5 = \frac{1 \times 5}{6} = \frac{5}{6} \Omega \)

(Replacing current source by its internal impedance i.e. open circuit)
4. For the following circuit find the value of \( v_{TH} \) and \( R_{TH} \)

\[ \begin{align*}
\text{Ans.} & \quad \text{To calculate } R_{th} \text{, deactivate all independent sources i.e, replace them by their internal impedances.} \\
R_{th} &= 30 + 25 = 55 \Omega \\
v_{th} &= \text{voltage at node P} \\
&= 5 \times 30 + 5 \\
&= 155 \text{ V}
\end{align*} \]

5. In the following circuit find its Thevenin and Norton equivalent

\[ \begin{align*}
\text{Ans.} & \quad v_{th} = 4 + 2 \times 2 \\
&= 8 \text{V}
\end{align*} \]
\[ R_{th} = 2 + 3 = 5 \Omega \]

Short circuit current is due to voltage source (4 Volts) and current source (2 Amp.)

Using superposition

Short circuit current due to voltage source = \( \frac{4}{5} \)

Short circuit current due to current source = \( \frac{2 \times 2}{2 + 3} = \frac{4}{5} \)

\[ I_{sc} = \frac{4}{5} + \frac{4}{5} = \frac{8}{5} \text{ amp} \]

Thevenin’s and Norton’s equivalents are shown below:
6. For the following circuit find the value of $v_{TH}$ and $R_{TH}$

Ans. Circuit does not have any independent source, So, $V_{th} = 0$.

For finding $R_{th}$ connect test voltage source $V_{test}$ at terminals XY supplying 1A

$i_1 = -1A$

Writing node equation at node P

$$\frac{V_{test}}{4} + \frac{V_{test}}{6} - 3i_1 + i_1 = 0$$

$5V_{test} = -6i_1 = 6$

$V_{test} = \frac{6}{5}$

$R_{th} = \frac{V_{test}}{1} = \frac{6}{5} = 1.2\Omega$
7. In the circuit shown in following figure find the value \(v_{TH}\) and \(R_{TH}\)

\[
\begin{array}{c}
\text{Ans. By source transformation} \\
\end{array}
\]

\[
V_{th} = v_1 \\
0.5v_1 + 4 = v_1 \\
0.5v_1 - v_1 = -4 \\
v_1 = \frac{4}{0.5} = 8 \\
V_{th} = 8 \text{ volts} \\
R_{th} = \frac{V_{th}}{i_{sc}}, \text{ } i_{sc} \text{ is short circuit current.} \\
\text{By putting short across } XY, v_1 = 0, \text{ so dependent voltage source } 0.5v_1 = 0, \text{ } i_{sc} = \frac{4}{5} \text{ so } R_{th} = \frac{V_{th}}{i_{sc}} = \frac{8}{4/5} = 10 \Omega
\]
8. In the following circuit find the effective resistance faced by the voltage source

\[
\text{Ans. } \quad i = \frac{v_s}{4} + \frac{i}{4}
\]

\[
4i = V_s + i
\]

\[
3i = V_s
\]

\[
i = \frac{v_s}{3}
\]

The effective resistance faced by the voltage source is 3Ω

9. Find the value of R (in ohms) for maximum power transfer in the network shown in the figure.

\[
\text{Ans. } \quad \text{For maximum power transfer } R_L = R_S \text{ (resistance looking into the network)}
\]

Replacing independent sources by their internal impedances.
\[ R_S = 5 \parallel 20 + 4 \]
\[ = 4 + 4 \]
\[ = 8 \Omega \]
\[ R = 8 \Omega \]

10. Find the Thevenin equivalent impedance \( Z_{th} \) between the nodes P and Q in the following circuit

Ans. For finding \( Z_{th} \) deactivate all the active sources

\[ Z_{th} \text{ (Between P & Q nodes)} \]
\[ (s + 1) \| \left( 1 + \frac{1}{s} \right) \]
\[ = 1 \Omega \]

11. Find the value of \( R_L \) such that the power transferred to \( R_L \) is maximum.

Answer: For maximum power transfer \( R_L = R_{th} \)

\[ R_{th} = 10 \| 10 + 10 \]
\[ = 5 + 10 \]
\[ = 15 \Omega \]
\[ R_L = 15 \Omega \]

12. A source \( v_s(t) = V \cos 100 \pi t \) has an internal impedance of \((4+j3) \Omega\). If an purely resistive load connected to this source has to extract the maximum power out of the source, find its value

Answer: For pure resistive load \( R_L \) to extract the maximum power
\[ R_L = \sqrt{R_S^2 + X_S^2} \]
\[ = \sqrt{4^2 + 3^2} \]
\[ = 5 \Omega \]