Two Port Networks

1. Two Two-port networks are connected in cascade. The combination is to be represented as a single two-port network, by multiplying the individual
   (a) $z$-parameter matrices
   (b) $h$-parameter matrices
   (c) $y$-parameter matrices
   (d) ABCD parameter matrices

   [GATE 1991: 2 Marks]

   **Soln.** ABCD parameters relate the voltage and current at one port to voltage and current at the other port

   Option (d)

2. For a 2-port network to be reciprocal,
   (a) $z_{11} = z_{22}$
   (b) $y_{21} = y_{12}$
   (c) $h_{21} = -h_{12}$
   (d) $AD - BC = 0$

   [GATE 1992: 2 Marks]

   **Soln.** $y_{21} = y_{12}$

   $h_2 = -h_{12}$

   Option (b) & (c)

3. The condition, that a 2-port network is reciprocal, can be expressed in terms of its ABCD parameters as……

   [GATE 1994: 1 Mark]

   **Soln.** $AD - BC = 1$
4. In the circuit of figure, the equivalent impedance seen across terminals A, B is

\[
Z_{eq} = (2||4) + (2||4)
\]

\[
= \frac{4}{3} + \frac{4}{3} = \frac{8}{3}
\]

Option (b) 

Soln. The product of the opposite arms are equal, so the bridge is balanced.

The point a and b are at the same potential

5. The short-circuit admittance matrix of a two-port network is

\[
\begin{bmatrix}
0 & -1/2 \\
1/2 & 0
\end{bmatrix}
\]

The two-port network is

(a) non-reciprocal and passive
(b) non-reciprocal and active
(c) reciprocal and passive
(d) reciprocal and active

[GATE 1997: 2 Marks]

[GATE 1998: 1 Marks]
Soln. \( y_{12} \neq y_{21} \) so the network is nonreciprocal. And active networks are nonreciprocal

Option (b)

6. A 2-port network is shown in the figure. The parameter \( h_{21} \) for this network can be given by

\[
\begin{align*}
I_1 & = h_{11} I_1 + h_{12} V_2 \\
I_2 & = h_{21} I_1 + h_{22} V_2 \\
h_{21} & = \frac{I_1}{I_2} |_{V_2=0} \\
V_2 & = RI_2 + R(I_1 + I_2) \\
\text{Or} \ 2RI_2 + RI_1 & = V_2 \\
\text{When} \ V_2 = 0, \ 2RI_2 + RI_1 & = 0 \\
\text{So,} \ \frac{I_2}{I_1} & = \frac{-R}{2R} = -\frac{1}{2} \\
\text{Option (a)}
\end{align*}
\]
7. The admittance parameter \( Y_{12} \) in the 2-port network in figure is

\[
E_1 \quad 5\Omega \quad I_1 \quad 20\Omega \quad I_2 \quad E_2
\]

(a) -0.2 nho
(b) 0.1 mho
(c) -0.05 mho
(d) 0.05 mho

[SOLN] \( y_{12} = -\frac{1}{20} = -0.05 \text{ mhos} \)

Option (c)

8. The Z parameters \( Z_{11} \) and \( Z_{21} \) for the 2-port network in the figure are

\[
I_1 \quad 2\Omega \quad I_2
\]

(a) \( Z_{11} = \frac{-6}{11} \Omega, Z_{21} = \frac{16}{11} \Omega \)
(b) \( Z_{11} = \frac{6}{11} \Omega, Z_{21} = \frac{4}{11} \Omega \)
(c) \( Z_{11} = \frac{6}{11} \Omega, Z_{21} = \frac{-16}{11} \Omega \)
(d) \( Z_{11} = \frac{4}{11} \Omega, Z_{21} = \frac{4}{11} \Omega \)

[SOLN] For z – parameters

\[
E_1 = Z_{11}I_1 + Z_{12}I_2
\]

\[
E_2 = Z_{21}I_1 + Z_{22}I_2
\]

Writing KVL in LHS loop
\[ E_1 = 2I_1 + 4I_1 + 4I_2 - 10E_1 \]

Or \[ 11E_1 = 6I_1 + 4I_2 \quad (I) \]

\[ Z_{11} = \frac{E_1}{I_1} \bigg|_{I_2 = 0} = \frac{6}{11} \Omega \]

\[ Z_{12} = \frac{E_1}{I_2} \bigg|_{I_1 = 0} = \frac{4}{11} \Omega \]

Writing KVL in RHS Loop

\[ E_2 = 4(I_1 + I_2) - 10E_1 \quad (II) \]

Substituting \( E_1 = \frac{6I_1 + 4I_2}{11} \) in equation \( (III) \)

\[ E_2 = 4(I_1 + I_2) - \frac{10(6I_1 + 4I_2)}{11} \]

\[ Z_{21} = \frac{E_2}{I_1} \bigg|_{I_2 = 0} = \frac{-16}{11} \Omega \]

Option (c)

9. The impedance parameters \( Z_{11} \) and \( Z_{12} \) of the two-port network in the figure are

- (a) \( Z_{11} = 2.75 \Omega \) and \( Z_{12} = 0.25 \Omega \)
- (b) \( Z_{11} = 3 \Omega \) and \( Z_{12} = 0.5 \Omega \)
- (c) \( Z_{11} = 3 \Omega \) and \( Z_{12} = 0.25 \Omega \)
- (d) \( Z_{11} = 2.25 \Omega \) and \( Z_{12} = 0.5 \Omega \)

[GATE 2003: 2 Marks]
Soln. Using $\Delta - Y$ conversion, the circuit reduces to

\[ Z_{11} = Z_1 + Z_3 \]
\[ = 2.5 + 0.25 \]
\[ = 2.75 \Omega \]
\[ Z_{12} = Z_3 = 0.25 \Omega \]

Option (a)

10. The $h$ parameter of the circuit shown in the figure are

\[ \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]

(a) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$

(c) $\begin{bmatrix} 30 & 20 \\ 20 & 30 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 1 \\ 10 & 0.05 \end{bmatrix}$

[GATE 2005: 2 Marks]
Soln. \[ V_1 = h_{11}I_1 + h_{12}V_2 \]
\[ I_2 = h_{21}I_1 + h_{22}V_2 \]

Writing KVL in LHS and RHS Loop

\[ V_1 = 10I_1 + 20(I_1 + I_2) \] \((i)\)
\[ V_2 = 20(I_2 + I_1) \] \((ii)\)

Or \[ V_1 = 10I_1 + V_2 \]

\[ h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 10 \]
\[ h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = -1 \]
\[ h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = 1 \]
\[ h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{20} = 0.05 \Omega \]

Option (d)

11. In the two network shown in the figure below \(Z_{12}\) and \(Z_{21}\) are, respectively
Soln.

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]
\[ V_2 = Z_{21} I_1 + Z_{22} I_2 \]

\[ Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \]

When \( I_1 = 0, V_1 = 0 \), so \( Z_{12} = 0 \)

\[ Z_{21} = \left. \frac{V_2}{I_1} \right|_{V_2=0} \]

When \( I_2 = 0, V_2 = -\beta I_1 r_0 \)

\[ Z_{21} = -\beta r_0 \]

Option (b)
12. For the two-port network shown, the short-circuit admittance parameter matrix is 

\[
\begin{bmatrix}
4 & -2 \\
-2 & 4
\end{bmatrix}
\]

(a) \[\frac{1}{2} \begin{bmatrix} 1 & -0.5 \\ 0.5 & 4 \end{bmatrix} S \]
(b) \[\begin{bmatrix} 1 & -0.5 \\ -0.5 & 4 \end{bmatrix} S \]
(c) \[\begin{bmatrix} 1 & -0.5 \\ 0.5 & 4 \end{bmatrix} S \]
(d) \[\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} S\]

[\text{GATE 2010: 1 Mark}]

\textbf{Soln.} The short circuit admittance parameters of a two port \( \pi \) network:

\[
y_{11} = y_a + y_b = \frac{1}{0.5} + \frac{1}{0.5} = 4 \Omega
\]

\[
y_{12} = y_{21} = -y_b = -\frac{1}{0.5} = -2 \Omega
\]

\[
y_{22} = y_b + y_c = \frac{1}{0.5} + \frac{1}{0.5} = 4 \Omega
\]

Option (a)

13. If the scattering matrix [S] of a two port network is

\[
[S] = \begin{bmatrix}
0.2 \angle 0^\circ & 0.9 \angle 0^\circ \\
0.9 \angle 0^\circ & 0.1 \angle 0^\circ
\end{bmatrix}
\]

Then the network is

(a) Lossless and reciprocal
(b) Lossless but not reciprocal
(c) Not lossless but reciprocal
(d) Neither lossless not reciprocal

[\text{GATE 2010: 1 Mark}]

\textbf{Soln.} For the reciprocal network \(|S_{12}| = |S_{21}|\)

For a loss less network \(|S_{11}|^2 + |S_{12}|^2 = 1\)

\[(0.2)^2 + (0.9)^2 \neq 1\]
The network is lossy and reciprocal.

Option (c)

14. In the circuit shown below, the network N is described by the following Y matrix:

\[
Y = \begin{bmatrix}
0.1S & -0.01S \\
0.01S & 0.1S
\end{bmatrix}
\]

The voltage gain \( \frac{V_2}{V_1} \) is

(a) \( \frac{1}{90} \)  
(b) \( -\frac{1}{90} \)  
(c) \( -\frac{1}{99} \)  
(d) \( -\frac{1}{11} \)  

[GATE 2011: 2 Marks]

Soln.

\[
I_2 = y_{21}V_1 + y_{22}V_2
\]

\[
= 0.01V_1 + 0.1V_2 - - - - - - (i)
\]

\[
V_2 = -I_2R_L = -100I_2
\]

\[
I_2 = \frac{-V_2}{100}
\]

Substituting the value of \( I_2 \) in equation (i)

\[
\frac{-V_2}{100} = 0.01V_1 + 0.1V_2
\]

Or \( \frac{V_2}{V_1} = -\frac{1}{11} \)

Option (d)